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A_∞-categories, WS 16/17

Sheet 7

Problem 17 Let $p > 0$ be a prime.

Let $P \in \mathrm{Ob}\,\mathrm{C}(\mathbf{F}_p\mathrm{C}_p\text{-Mod})$ be the projective resolution of the trivial $\mathbf{F}_p\mathrm{C}_p$-module as found in Problem 9.(1).

Let $\underline{X} := (P)$, so that $\underline{X}$ has P as its only tuple entry.

Let $A := \mathrm{Hom}_{\mathbf{F}_p\mathrm{C}_p}(\underline{X})$ be the regular differential graded category, i.e. differential graded algebra over $\mathbf{Z} = \mathbf{Z} \times [1,1]^{\times 2}$.

Recall from Problem 9 and Problem 14 that we have calculated the $\mathbf{Z}$-graded module $\mathrm{H}A$, i.e. that we know $\mathbf{F}_p$-linear generators for its graded pieces.

Find a minimal A_3-structure $(\tilde{m}_1, \tilde{m}_2, \tilde{m}_3)$ on $\mathrm{H}A$ and a quasiisomorphism $(q_1, q_2, q_3) : \mathrm{H}A \to A$ of A_3-algebras.

Problem 18 Suppose R to be a field.

Let $\mathcal{Z}$ be a grading category. Let $n \in [1, \infty]$. Let A be a unital A_n-algebra over $\mathcal{Z}$.

Consider the shift-graded linear residue class map $\mathrm{Z}A \xrightarrow{\rho} \mathrm{H}A$ of degree 0.

Show that there exists a shift graded linear map $\mathrm{Z}A \xleftarrow{\sigma} \mathrm{H}A$ of degree 0 such that $\sigma\rho = \mathrm{id}_{\mathrm{H}A}$ and such that $(1_X\rho)\sigma = 1_X$ for $X \in \mathrm{Ob}(\mathcal{Z})$.