Problem 15 Let $\mathcal{Z}$ be a grading category.
Let $L$ and $M$ be $\mathcal{Z}$-graded modules.
Let $L \xrightarrow{f} M$ be a shift-graded linear map of degree $a \in \mathbb{Z}$.

(1) Let $K$ be the $\mathcal{Z}$-graded module defined by $K^z := \text{Kern}(L^z \xrightarrow{f} M^{z[a]})$ for $z \in \text{Mor}(\mathcal{Z})$.
Let $K \xrightarrow{i} L$ be the shift-graded linear map given by inclusion, which is of degree 0.
Suppose given a $\mathcal{Z}$-graded module $T$ and a shift-graded linear map $T \xrightarrow{t} L$ of degree $d$ such that $tf = 0$.
Show that there exist a unique shift-graded linear map $T \xrightarrow{\tilde{t}} K$ of degree $d$ such that $\tilde{ti} = t$.

(2) Suppose $R$ to be a field. Suppose $f$ to be piecewise surjective.
Show that there exists a piecewise injective shift-graded linear map $L \xleftarrow{g} M$ of degree $-a$ such that $gf = \text{id}_M$.

Problem 16 Let $\mathcal{Z}$ be a grading category. Let $n \in \mathbb{Z}_{\geq 1}$.
Let $K_i$, $L_i$ and $M_i$ be $\mathcal{Z}$-graded modules for $i \in [1, n]$.
Let $K_i \xrightarrow{u_i} L_i$ be a shift-graded linear map of degree $c_i \in \mathbb{Z}$ for $i \in [1, n]$.
Let $L_i \xrightarrow{f_i} M_i$ be a piecewise surjective shift-graded linear map of degree $a_i \in \mathbb{Z}$ for $i \in [1, n]$.

(1) Show that $\bigotimes_{i \in [1, n]} f_i$ is piecewise surjective.

(2) For $i \in [1, n]$, suppose that $K_i \xrightarrow{u_i} L_i \xrightarrow{f_i} M_i$ to be exact at $L_i$,
i.e. suppose $K_i^{z[-c_i]} \xrightarrow{u_i} L_i^z \xrightarrow{f_i} M_i^{z[a_i]}$ to be exact at $L_i^z$ for $z \in \text{Mor}(\mathcal{Z})$.
Suppose given a $\mathcal{Z}$-graded module $T$ and a shift-graded linear map $\bigotimes_{i \in [1, n]} L_i \xrightarrow{t} T$ of degree $d$.
Suppose that $(\text{id}^\otimes j^{-1} \otimes u_j \otimes \text{id}^\otimes n-j)t = 0$ for $j \in [1, n]$.
Show that there exists a unique shift-graded linear map $\bigotimes_{i \in [1, n]} M_i \xrightarrow{\tilde{t}} T$ of degree $d - \sum_{i \in [1, n]} a_i$ such that $(\bigotimes_{i \in [1, n]} f_i)\tilde{t} = t$. 

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