Recent developments

Regarding Chapter 1:

Regarding Chapter 2:

Regarding Chapter 3:

Regarding Chapter 4:

Regarding Chapter 5:

[CPP: 55.38] can be improved: Two closed Baer subplanes always have a point and a line in common, not only if they are the fixed point sets of involutions. See R. Löwen [1998b].

Regarding Chapter 6:

[CPP: 65.2] can be improved: it suffices to assume that $\dim\Sigma \geq C_\ell - 2$. The following additional argument is then required in step 1) of the *proof*:
If B is contained in a line L through x, then $\Sigma_x = \Sigma_{x,L}$ and, for a suitable $\sigma \in \Sigma$, the stabilizer Σ_{x,x^σ} fixes a triangle, but $\dim\Sigma_{x,x^\sigma} \geq C_\ell - 2 - 4\ell$ is too large.

Regarding Chapter 7:

Regarding Chapter 8:

8.a.
The following results improve [CPP: 87.1]:

If $\mathbf{P}$ *is a* 16*-dimensional plane, and if* $\dim\Delta \geq 27$*, then* Δ *is a Lie group.*
See B. Priwitzer - H. Salzmann [1998].

Irrespective of connectedness, the automorphism groups of a 16*-dimensional projective plane is a Lie group, if it has dimension at least* 29; see H. Salzmann [1999a].

8.b.
Regarding [CPP: 87.2] we remark that the same results have been obtained in B. Priwitzer [1997] and B. Priwitzer [1998] under the weaker hypothesis that $\dim\Delta > 28$ (instead of 30).

8.c.
[CPP: 87.3] is true for $\dim\Delta \geq 31$ instead of 35. See H. Salzmann [1998].

8.d.
ad [CPP: 87.4]: the arguments on p.587 can be improved as follows:
Theorem. *If* Δ *is not semi-simple, and if* $\dim\Delta \geq 33$*, then, up to duality,* Δ *has a minimal normal subgroup* $\Theta \cong \mathbf{R}^t$ *consisting of axial collineations with common axis* W*.*
Either $\Theta \leq \Delta_{[a,W]}$ *is a group of homologies and* $t = 1$*, or* Θ *is contained in the group* $\mathrm{T} = \Delta_{[W,W]}$ *of elations with axis* W*.*
For full proofs see H. Salzmann [1999b].

8.e.
The results described in Section 87 have been extended, as follows.
Theorem. *If* $\dim\Delta \geq 35$ *and if* Δ *fixes exactly one line* W *and no point, then* $\mathbf{P}$ *is a translation plane.*
Recently, Hähl and Löwe have determined explicitly all translation planes having a group Δ as in this theorem. In particular, their work implies the following:
Corollary. *Under the assumptions of the Theorem, either* $\mathbf{P} \cong \mathbf{P}_2\mathbf{O}$*, or* $\dim\Delta = 35$ *and the stabilizer of an affine point has an 18-dimensional semisimple commutator group* Υ *isomorphic to one of the groups* $\mathrm{SL}_2\mathbf{H}\cdot\mathrm{SU}_2\mathbf{C}$ *or* $\mathrm{SU}_4\mathbf{C}\cdot D$ *with* $D = \mathrm{SU}_2\mathbf{C}$ *or* $D = \mathrm{SL}_2\mathbf{R}$*.*
If Δ *fixes two distinct points, and if* $\dim\Delta = 39$*, then* $\mathbf{P}$ *is a translation plane over a perturbation of the octonions* [CPP: 82.5b]; see H. Salzmann [1999b].

8.f.
The following results are obtained in H. Salzmann [1999d]:
Lemma. *Assume that* $\dim\Delta \geq 29$*. If* Δ *fixes no point and no line, then* $\mathbf{P}$ *is classical, or* $\Delta' \cong \mathrm{SL}_3(\mathbf{H})$ *and* $\mathbf{P}$ *is a Hughes plane. If* Δ *fixes exactly one element, then* Δ *has a normal vector subgroup.*
Theorem. *If* $\dim\Delta \geq 35$ *and if* Δ *fixes exactly one line* W *and no point, then* $\mathbf{P}$ *is a translation plane.*
Corollary. *If* $\dim\Delta \geq 38$ *and if* Δ *fixes exactly one element (point or line), then* $\mathbf{P}$ *is isomorphic to the octonion plane.*

8.g.
Moreover, the following is true (H. Salzmann [2000–2003]):

Theorem. *If* $\dim\Delta \geq 34$ *and if* Δ *fixes exactly* 2 *points* u, v *and the line* uv*, then the group* T *of translations with axis* uv *is at least* 15*-dimensional. Either* Δ *has a subgroup* $Y \cong \mathrm{Spin}_7\mathbf{R}$ *and* $\dim\Delta \geq 36$*, or* T *is transitive, a maximal semi-simple subgroup of* Δ *is isomorphic to* $\mathrm{SU}_4\mathbf{C} \cong \mathrm{Spin}_6\mathbf{R}$*, and* $\dim\Delta = 34$*.*

Theorem. *If* Δ *fixes exactly* 2 *points* u, v *and* 2 *lines* $W = uv$ *and* $Y = av$*, then the translation group* $T = \Delta_{[v,W]}$ *is transitive, the complement* Δ_a *of* T *has a compact commutator group* $\Phi \cong \mathrm{Spin}_8\mathbf{R}$*, and* $\dim\Delta \geq 36$*. If even* $\dim\Delta \geq 38$*, then* $\mathbf{P}$ *is the classical Moufang plane.*

Theorem. *If* $\dim\Delta \geq 32$ *and* Δ *has (at least)* 3 *fixed points, then* Δ *contains a transitive translation group* T*. Either* $\dim\Delta = 32$ *and a maximal semi-simple subgroup* Ψ *of* Δ *is isomorphic to* $\mathrm{SU}_4\mathbf{C}$*, or* $\dim\Delta \geq 37$ *and* $\mathbf{P}$ *is the classical Moufang plane.*

Regarding Chapter 9: