

Table of Laplace Transforms

Operations

$F(s)$	$f(t)$
$F(as) \quad (a > 0)$	$\frac{1}{a} f\left(\frac{t}{a}\right)$
$F(s + \alpha)$	$e^{-\alpha t} f(t)$
$F(as + \beta) \quad (a > 0, \beta \text{ complex})$	$\frac{1}{a} e^{-\frac{\beta}{a} t} f\left(\frac{t}{a}\right)$
$e^{-as} F(s) \quad (a \geq 0)$	$\begin{cases} f(t - a) & \text{for } t \geq a \\ 0 & \text{for } t < a \end{cases} = f(t - a) u(t - a)$
$F'(s)$	$-t f(t)$
$F^{(n)}(s)$	$(-t)^n f(t)$
$sF(s) - f(0^+)$	$f'(t)$
$sF(s)$	$Df(t)$
$s^n F(s) - \sum_{k=0}^{n-1} f^{(k)}(0^+) s^{n-k-1}$	$f^{(n)}(t)$
$s^n F(s)$	$D^n f(t)$
$\frac{1}{s} F(s)$	$\int_0^t f(\tau) d\tau = f(t) * 1$
$F(\sqrt{s})$	$\int_0^\infty \psi(\tau, t) f(\tau) d\tau$
$F_1(s) \cdot F_2(s)$	$\int_0^t f_1(\tau) f_2(t - \tau) d\tau = f_1(t) * f_2(t)$
$\frac{1}{2\pi i} \int_{x-i\infty}^{x+i\infty} F_1(\sigma) F_2(s - \sigma) d\sigma$	$f_1(t) \cdot f_2(t)$

Nr.	$F(s)$	$f(t)$
1	1	$\delta(t)$
2	s^n ($n = 1, 2, \dots$)	$\delta^{(n)}(t)$
3	s^a ($a > 0, a \neq 1, 2, \dots$)	$\text{Pf} \frac{t^{-a-1}}{\Gamma(-a)} u(t)$ $= -\text{Pf} \frac{1}{\pi} \Gamma(a+1) \sin \pi a \frac{1}{t^{a+1}} u(t)$
4	$\frac{1}{s}$	$u(t) = 1$ for $t > 0$, $= 0$ for $t < 0$
5	$\frac{1}{s - \alpha}$	$e^{\alpha t}$
6	$\frac{1}{1 + Ts}$	$\frac{1}{T} e^{-\frac{t}{T}}$
7	$\frac{1}{s^2}$	t
8	$\frac{1}{(s - \alpha)^2}$	$t e^{\alpha t}$
9	$\frac{1}{s(s - \alpha)}$	$\frac{1}{\alpha} (e^{\alpha t} - 1)$
10	$\frac{1}{s(1 + Ts)}$	$1 - e^{-\frac{t}{T}}$
11	$\frac{1}{(s - \alpha)(s - \beta)}$ ($\alpha \neq \beta$)	$\frac{e^{\alpha t} - e^{\beta t}}{\alpha - \beta}$
12	$\frac{1}{(1 + Ts)^2}$	$\frac{1}{T^2} t e^{-\frac{t}{T}}$
13	$\frac{1}{(1 + as)(1 + bs)}$ ($a \neq b$)	$\frac{e^{-\frac{t}{a}} - e^{-\frac{t}{b}}}{a - b}$
14	$\frac{a}{s^2 + a^2}$	$\sin at$
15	$\frac{a}{s^2 - a^2}$	$\sinh at$
16	$\frac{1}{s^2 + c_1 s + c_0}$ ($c_0 - \frac{c_1^2}{4} = D$)	$\begin{cases} \frac{1}{\sqrt{-D}} e^{-\frac{c_1}{2} t} \sinh \sqrt{-D} t & (D < 0) \\ \frac{1}{\omega} e^{-\frac{c_1}{2} t} \sin \omega t & (D > 0, \sqrt{-D} = i\omega) \\ t e^{-\frac{c_1}{2} t} & (D = 0) \end{cases}$
17	$\frac{s}{(s - \alpha)^2}$	$(1 + \alpha t) e^{\alpha t}$

Nr.	$F(s)$	$f(t)$
18	$\frac{s}{(s-\alpha)(s-\beta)} \quad (\alpha \neq \beta)$	$\frac{\alpha e^{\alpha t} - \beta e^{\beta t}}{\alpha - \beta}$
19	$\frac{s}{s^2 + a^2}$	$\cos at$
20	$\frac{s}{s^2 - a^2}$	$\cosh at$
21	$\frac{s \sin b + a \cos b}{s^2 + a^2}$	$\sin (at + b)$
22	$\frac{s \cos b - a \sin b}{s^2 + a^2}$	$\cos (at + b)$
23	$\frac{1}{s^n} \quad (n = 1, 2, \dots)$	$\frac{1}{(n-1)!} t^{n-1}$
24	$\frac{1}{(s-\alpha)^n} \quad (n = 1, 2, \dots)$	$\frac{1}{(n-1)!} t^{n-1} e^{\alpha t}$
25	$\frac{1}{s} \left(1 - \frac{1}{s}\right)^n$	$L_n(t) = \frac{e^t}{n!} \frac{d^n}{dt^n} (t^n e^{-t})$ (Laguerre Polynomials)
26	$\frac{1}{s^\alpha} \quad (\alpha \text{ arbitrarily real})$	$\begin{cases} \frac{t^{\alpha-1}}{\Gamma(\alpha)} u(t) & \text{for } \alpha > 0 \\ \text{Pf } \frac{t^{\alpha-1}}{\Gamma(\alpha)} u(t) & \text{for } \alpha < 0, \alpha \neq -1, -2, \dots \\ \delta^{(n)}(t) & \text{for } \alpha = -n = 0, -1, -2, \dots \end{cases}$
27	$\frac{1}{\sqrt{s}}$	$\frac{1}{\sqrt{\pi t}}$
28	$\frac{1}{\sqrt{s+\alpha}}$	$\frac{e^{-\alpha t}}{\sqrt{\pi t}}$
29	$\frac{1}{\sqrt{s+a}}$	$\frac{1}{\sqrt{\pi t}} - a e^{a^2 t} \operatorname{erfc}(a\sqrt{t})$
30	$\frac{1}{s^{3/2}}$	$2 \sqrt{\frac{t}{\pi}}$
31	$\frac{1}{s^{n+\frac{1}{2}}}$	$\frac{4^n n!}{(2n)! \sqrt{\pi}} t^{n-\frac{1}{2}}$
32	$\frac{1}{\sqrt{s^2+a^2}}$	$J_0(at)$
33	$\frac{1}{(s^2+a^2)^{\nu+\frac{1}{2}}} \quad (\Re \nu > -\frac{1}{2})$	$\frac{\sqrt{\pi}}{\Gamma(\nu+\frac{1}{2})} \left(\frac{t}{2a}\right)^\nu J_\nu(at)$
34	$(\sqrt{s^2+\alpha^2} - s)^\nu \quad (\Re \nu > 0)$	$\frac{\nu \alpha^\nu}{t} J_\nu(\alpha t)$

Nr.	$F(s)$	$f(t)$
35	$\frac{(\sqrt{s^2 + \alpha^2} - s)^\nu}{\sqrt{s^2 + \alpha^2}} \quad (\Re \nu > -1)$	$\alpha^\nu J_\nu(\alpha t)$
36	$\sqrt{s - \alpha} - \sqrt{s - \beta}$	$\frac{1}{2t\sqrt{\pi t}} (e^{\beta t} - e^{\alpha t})$
37	$\frac{\log s}{s}$	$-\log t - C$
38	$\log \frac{s + a}{s - a}$	$\frac{2}{t} \sinh at$
39	$\log \frac{s - a}{s - b}$	$\frac{e^{bt} - e^{at}}{t}$
40	$\log \frac{s^2 + a^2}{s^2 + b^2}$	$\frac{2}{t} (\cos bt - \cos at)$
41	$\log s + C$	$-\text{Pf} \frac{1}{t} u(t)$
42	$s(\log s + C - 1)$	$\text{Pf} \frac{1}{t^2} u(t)$
43	$s^n (\log s - \psi(n + 1))$ ($n = 0, 1, \dots$)	$-(-1)^n n! \text{Pf} \frac{1}{t^{n+1}} u(t)$
44	$\log \sqrt{s^2 + a^2} + C$	$\text{Pf} \frac{\cos at}{t} u(t)$
45	$e^{-Ts} \quad (T > 0)$	$\delta(t - T)$
46	$\frac{e^{-Ts}}{s} \quad (T > 0)$	$u(t - T)$
47	$\frac{1}{s} e^{-\frac{a^2}{4s}}$	$J_0(a\sqrt{t})$
48	$\frac{1}{s^{\nu+1}} e^{-\frac{a^2}{4s}} \quad (\Re \nu > -1)$	$\left(\frac{2}{a}\right)^\nu t^{\frac{\nu}{2}} J_\nu(a\sqrt{t})$
49	$e^{-a\sqrt{s}} \quad (a > 0)$	$\psi(a, t) = \frac{a}{2\sqrt{\pi} t^{3/2}} e^{-\frac{a^2}{4t}}$
50	$\frac{1}{\sqrt{s}} e^{-a\sqrt{s}} \quad (a \geq 0)$	$\chi(a, t) = \frac{1}{\sqrt{\pi t}} e^{-\frac{a^2}{4t}}$
51	$\psi(a, s)$	$\frac{\sin a\sqrt{t}}{\pi}$
52	$\chi(a, s)$	$\frac{\cos a\sqrt{t}}{\pi\sqrt{t}}$
53	$\frac{1}{s} e^{-a\sqrt{s}} \quad (a \geq 0)$	$\text{erfc} \frac{a}{2\sqrt{t}} = \frac{2}{\sqrt{\pi}} \int_{a/2\sqrt{t}}^{\infty} e^{-u^2} du$

Nr.	$F(s)$	$f(t)$
54	$\frac{1}{\sqrt{s}} \sin \frac{a}{s}$	$\frac{\sinh \sqrt{2at} \sin \sqrt{2at}}{\sqrt{\pi t}}$
55	$\frac{1}{\sqrt{s}} \cos \frac{a}{s}$	$\frac{\cosh \sqrt{2at} \cos \sqrt{2at}}{\sqrt{\pi t}}$
56	$\frac{1}{\sqrt{s}} e^{-\sqrt{as}} \sin \sqrt{as}$	$\frac{1}{\sqrt{\pi t}} \sin \frac{a}{2t}$
57	$\frac{1}{\sqrt{s}} e^{-\sqrt{as}} \cos \sqrt{as}$	$\frac{1}{\sqrt{\pi t}} \cos \frac{a}{2t}$
58	$e^{\frac{s^2}{4}} \operatorname{erfc} \frac{s}{2}$	$\frac{2}{\sqrt{\pi}} e^{-t^2}$
59	$\operatorname{arctg} \frac{a}{s}$	$\frac{\sin at}{t}$
60	$\frac{\sinh x \sqrt{s}}{\sinh l \sqrt{s}} \quad (x < l)$	$\frac{1}{l} \frac{\partial}{\partial x} \vartheta_3 \left(\frac{l-x}{2l}, \frac{t}{l^2} \right)$ $= \sum_{n=-\infty}^{\infty} \psi(2nl + l - x, t)$ $= -\frac{2\pi}{l^2} \sum_{n=1}^{\infty} (-1)^n n \sin n \frac{\pi}{l} x e^{-n^2 \frac{\pi^2}{l^2} t}$