t-structures and recollements

Yann Palu and Teruji Thomas

Isle of Skye 2010

after Beilinson, Bernstein and Deligne *Faisceaux pervers.* Analysis and topology on singular spaces, I (Luminy, 1981), Asterisque. 100. Soc. Math. France, Paris. pp. 5–171.

Setup

Let $\mathcal{C}$ be a triangulated category.
Given two strictly full subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ define $\mathcal{C}^{\leq n} = \mathcal{C}^{\leq 0}[-n]$ and $\mathcal{C}^{\geq n} = \mathcal{C}^{\geq 0}[-n]$

Example

$$\mathcal{C} = \text{homotopy category of complexes of} \cdots$$

$$\mathcal{C}^{\leq 0} \qquad \cdots \to C^{-2} \to C^{-1} \to C^{0} \to 0 \to 0 \to \cdots$$

$$\mathcal{C}^{\geq 0} \qquad \cdots \to 0 \to 0 \to C^{0} \to C^{1} \to C^{2} \to \cdots$$

Setup

Let $\mathcal{C}$ be a triangulated category.
Given two strictly full subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ define
$\mathcal{C}^{\leq n} = \mathcal{C}^{\leq 0}[-n]$ and $\mathcal{C}^{\geq n} = \mathcal{C}^{\geq 0}[-n]$

Example

$\mathcal{C}$ = homotopy category of complexes of $\cdots$

$$\mathcal{C}^{\leq 0} \qquad \cdots \to C^{-2} \to C^{-1} \to C^0 \longrightarrow 0 \longrightarrow 0 \longrightarrow \cdots$$

$$\mathcal{C}^{\geq 0} \qquad \cdots \longrightarrow 0 \longrightarrow 0 \longrightarrow C^0 \to C^1 \to C^2 \to \cdots$$

Setup

Let $\mathcal{C}$ be a triangulated category.
Given two strictly full subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ define
$\mathcal{C}^{\leq n} = \mathcal{C}^{\leq 0}[-n]$ and $\mathcal{C}^{\geq n} = \mathcal{C}^{\geq 0}[-n]$

Example

$\mathcal{C} =$ homotopy category of complexes of $\cdots$

$$\mathcal{C}^{\leq 0} \qquad \cdots \to C^{-2} \to C^{-1} \to C^0 \longrightarrow 0 \longrightarrow 0 \longrightarrow \cdots$$

$$\mathcal{C}^{\geq 0} \qquad \cdots \longrightarrow 0 \longrightarrow 0 \longrightarrow C^0 \to C^1 \to C^2 \to \cdots$$

Setup

Let $\mathcal{C}$ be a triangulated category.
Given two strictly full subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ define $\mathcal{C}^{\leq n} = \mathcal{C}^{\leq 0}[-n]$ and $\mathcal{C}^{\geq n} = \mathcal{C}^{\geq 0}[-n]$

Example

$$\mathcal{C} = \text{homotopy category of complexes of} \cdots$$

$$\mathcal{C}^{\leq 0} \qquad \cdots \to C^{-2} \to C^{-1} \to C^0 \to 0 \to 0 \to \cdots$$

$$\mathcal{C}^{\geq 0} \qquad \cdots \to 0 \to 0 \to C^0 \to C^1 \to C^2 \to \cdots$$

Setup

Let $\mathcal{C}$ be a triangulated category.
Given two strictly full subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ define $\mathcal{C}^{\leq n} = \mathcal{C}^{\leq 0}[-n]$ and $\mathcal{C}^{\geq n} = \mathcal{C}^{\geq 0}[-n]$

Example

$\mathcal{C}$ = homotopy category of complexes of $\cdots$

$$\mathcal{C}^{\leq n} \qquad \cdots \to C^{n-2} \to C^{n-1} \to C^{n} \longrightarrow 0 \longrightarrow 0 \longrightarrow \cdots$$

$$\mathcal{C}^{\geq n} \qquad \cdots \longrightarrow 0 \longrightarrow 0 \longrightarrow C^{n} \to C^{n+1} \to C^{n+2} \to \cdots$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0} X \longrightarrow X \longrightarrow \tau_{\geq 1} X \longrightarrow$ with $\tau_{\leq 0} X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1} X \in \mathcal{C}^{\geq 1}$.

Example

$$
\begin{array}{ccccccccccccccc}
\mathcal{C}^{\leq 0} & & \cdots & \to & C^{-2} & \to & C^{-1} & \to & C^{0} & \to & 0 & \to & 0 & \to & \cdots \\
\downarrow {\scriptstyle 0} & & & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\mathcal{C}^{\geq 1} & & \cdots & \to & 0 & \to & 0 & \to & 0 & \to & C^{1} & \to & C^{2} & \to & \cdots
\end{array}
$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0}X \longrightarrow X \longrightarrow \tau_{\geq 1}X \longrightarrow$ with $\tau_{\leq 0}X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1}X \in \mathcal{C}^{\geq 1}$.

Example

$$\begin{array}{ccccccccccccccc}
\mathcal{C}^{\leq 0} & & \cdots & \to & C^{-2} & \to & C^{-1} & \to & C^{0} & \to & 0 & \to & 0 & \to & \cdots \\
\downarrow & & & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\mathcal{C}^{\geq 1} & & \cdots & \to & 0 & \to & 0 & \to & 0 & \to & C^{1} & \to & C^{2} & \to & \cdots
\end{array}$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0} X \longrightarrow X \longrightarrow \tau_{\geq 1} X \longrightarrow$ with $\tau_{\leq 0} X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1} X \in \mathcal{C}^{\geq 1}$.

Example

$$\begin{array}{ccccccccccccc}
\mathcal{C}^{\leq 0} & & \cdots \to & C^{-2} & \to & C^{-1} & \to & C^{0} & \to & 0 & \to & 0 & \to \cdots \\
\downarrow 0 & & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & \\
\mathcal{C}^{\geq 1} & & \cdots \to & 0 & \to & 0 & \to & 0 & \to & C^{1} & \to & C^{2} & \to \cdots
\end{array}$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0}X \longrightarrow X \longrightarrow \tau_{\geq 1}X \longrightarrow$ with $\tau_{\leq 0}X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1}X \in \mathcal{C}^{\geq 1}$.

Example

$$\begin{array}{cccccccccccccc}
\mathcal{C}^{\leq 0} & & \cdots \to & C^{-2} & \to & C^{-1} & \to & C^{0} & \to & 0 & \to & 0 & \to & \cdots \\
\downarrow 0 & & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\mathcal{C}^{\geq 1} & & \cdots \to & 0 & \to & 0 & \to & 0 & \to & C^{1} & \to & C^{2} & \to & \cdots
\end{array}$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0}X \longrightarrow X \longrightarrow \tau_{\geq 1}X \longrightarrow$ with $\tau_{\leq 0}X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1}X \in \mathcal{C}^{\geq 1}$.

Example

$$\begin{array}{ccccccccccccccc}
\mathcal{C}^{\leq 0} & & \cdots & \to & C^{-2} & \to & C^{-1} & \to & C^{0} & \to & 0 & \to & 0 & \to & \cdots \\
\downarrow{\scriptstyle 0} & & & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \downarrow & & \\
\mathcal{C}^{\geq 1} & & \cdots & \to & 0 & \to & 0 & \to & 0 & \to & C^{1} & \to & C^{2} & \to & \cdots
\end{array}$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0}X \longrightarrow X \longrightarrow \tau_{\geq 1}X \longrightarrow$ with $\tau_{\leq 0}X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1}X \in \mathcal{C}^{\geq 1}$.

Example

$$\begin{array}{ll} \mathcal{C}^{\leq 0} & \cdots \to C^{-2} \to C^{-1} \to C^{0} \longrightarrow 0 \longrightarrow 0 \to \cdots \\ \big\downarrow & \\ \mathcal{C}^{\leq 1} & \cdots \to D^{-2} \to D^{-1} \to D^{0} \to D^{1} \to 0 \to \cdots \end{array}$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0}X \longrightarrow X \longrightarrow \tau_{\geq 1}X \longrightarrow$ with $\tau_{\leq 0}X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1}X \in \mathcal{C}^{\geq 1}$.

Example

$$\begin{array}{lccccccccccccc}
\mathcal{C}^{\leq 0} & \cdots & \to & C^{-2} & \to & C^{-1} & \to & C^{0} & \to & 0 & \to & 0 & \to & \cdots \\
\big\downarrow & & & & & & & & & & & & & \\
\mathcal{C}^{\leq 1} & \cdots & \to & D^{-2} & \to & D^{-1} & \to & D^{0} & \to & D^{1} & \to & 0 & \to & \cdots
\end{array}$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0} X \longrightarrow X \longrightarrow \tau_{\geq 1} X \longrightarrow$ with $\tau_{\leq 0} X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1} X \in \mathcal{C}^{\geq 1}$.

Example

$$
\begin{array}{ccccccccccccccc}
\tau_{\leq 0} X & & \cdots & \to & C^{-2} & \to & C^{-1} & \longrightarrow & \mathrm{Im}\, d^0 & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & \cdots \\
\downarrow & & & & \| & & \| & & \downarrow & & \downarrow & & \downarrow & & \\
X & & \cdots & \to & C^{-2} & \to & C^{-1} & \xrightarrow{d^0} & C^0 & \xrightarrow{d^1} & C^1 & \to & C^2 & \to & \cdots \\
\downarrow & & & & \downarrow & & \downarrow & & \downarrow & & \| & & \| & & \\
\tau_{\geq 1} X & & \cdots & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & C^0 / \mathrm{Ker}\, d^1 & \to & C^1 & \to & C^2 & & \cdots
\end{array}
$$

Definition of a t-structure

The subcategories $\mathcal{C}^{\leq 0}$, $\mathcal{C}^{\geq 0}$ form a t-structure on $\mathcal{C}$ if:

(i) $\mathrm{Hom}_{\mathcal{C}}(\mathcal{C}^{\leq 0}, \mathcal{C}^{\geq 1}) = 0$

(ii) $\mathcal{C}^{\leq 0} \subset \mathcal{C}^{\leq 1}$ and $\mathcal{C}^{\geq 0} \supset \mathcal{C}^{\geq 1}$

(iii) For all $X \in \mathcal{C}$, there is a triangle $\tau_{\leq 0}X \longrightarrow X \longrightarrow \tau_{\geq 1}X \longrightarrow$ with $\tau_{\leq 0}X \in \mathcal{C}^{\leq 0}$ and $\tau_{\geq 1}X \in \mathcal{C}^{\geq 1}$.

Example

$$\begin{array}{ccccccccccccccc}
\tau_{\leq 0}X & & \cdots & \to & C^{-2} & \to & C^{-1} & \longrightarrow & \mathrm{Im}\, d^0 & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & \cdots \\
\downarrow & & & & \| & & \| & & \hookdownarrow & & \downarrow & & \downarrow & & \\
X & & \cdots & \to & C^{-2} & \to & C^{-1} & \xrightarrow{d^0} & C^0 & \xrightarrow{d^1} & C^1 & \to & C^2 & \to & \cdots \\
\downarrow & & & & \downarrow & & \downarrow & & \twoheaddownarrow & & \| & & \| & & \\
\tau_{\geq 1}X & & \cdots & \longrightarrow & 0 & \longrightarrow & 0 & \longrightarrow & C^0 / \mathrm{Ker}\, d^1 & \to & C^1 & \to & C^2 & & \cdots
\end{array}$$

Crucial remark

A triangulated category may have several different t-structures.

- Shifting a t-structure: $(\mathcal{C}^{\leq n}, \mathcal{C}^{\geq n})$ is a t-structure.
- The category $D^b(\mathrm{Coh}\, P^1)$ is equivalent to $D^b(\mathrm{mod}\, A)$ where A is the Kronecker algebra.

Crucial remark

A triangulated category may have several different t-structures.

- Shifting a t-structure: $(\mathcal{C}^{\leq n}, \mathcal{C}^{\geq n})$ is a t-structure.
- The category $D^b(\text{Coh } P^1)$ is equivalent to $D^b(\text{mod } A)$ where A is the Kronecker algebra.

Crucial remark

A triangulated category may have several different t-structures.

- Shifting a t-structure: $(\mathcal{C}^{\leq n}, \mathcal{C}^{\geq n})$ is a t-structure.
- The category $D^b(\mathrm{Coh}\, P^1)$ is equivalent to $D^b(\mathrm{mod}\, A)$ where A is the Kronecker algebra.

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof

$$\begin{array}{ccccccc} & & & & Y & & \in \mathcal{C}^{\leq 0} \\ & & & & \downarrow & & \\ (\tau_{\geq 1}X)[-1] & \longrightarrow & \tau_{\leq 0}X & \longrightarrow & X & \longrightarrow & \tau_{\geq 1}X \end{array}$$

$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \xrightarrow{\simeq} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof

$$\begin{array}{ccccccc} & & & & Y & & \in \mathcal{C}^{\leq 0} \\ & & & & \downarrow & & \\ (\tau_{\geq 1}X)[-1] & \longrightarrow & \tau_{\leq 0}X & \longrightarrow & X & \longrightarrow & \tau_{\geq 1}X \end{array}$$

$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \xrightarrow{\simeq} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof

$$\begin{array}{ccccccc} & & & & Y & & \in \mathcal{C}^{\leq 0} \\ & & & & \downarrow & & \\ (\tau_{\geq 1}X)[-1] & \longrightarrow & \tau_{\leq 0}X & \longrightarrow & X & \longrightarrow & \tau_{\geq 1}X \end{array}$$

$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \xrightarrow{\simeq} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof

$$Y \in \mathcal{C}^{\leq 0}, \quad Y \longrightarrow X$$

$$(\tau_{\geq 1}X)[-1] \longrightarrow \tau_{\leq 0}X \longrightarrow X \longrightarrow \tau_{\geq 1}X$$

$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \xrightarrow{\simeq} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof

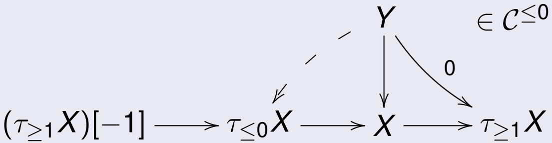
$$\begin{array}{ccccccc} & & & & Y & & \in \mathcal{C}^{\leq 0} \\ & & & & \downarrow & \searrow^{0} & \\ (\tau_{\geq 1}X)[-1] & \longrightarrow & \tau_{\leq 0}X & \longrightarrow & X & \longrightarrow & \tau_{\geq 1}X \end{array}$$

$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \xrightarrow{\simeq} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof



$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \xrightarrow{\simeq} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof

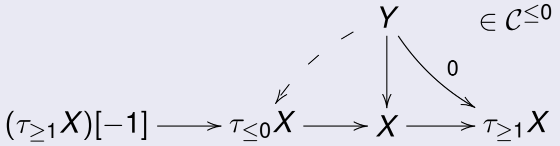
$$\begin{array}{ccccccc} & & & & Y & & \in \mathcal{C}^{\leq 0} \\ & & & \swarrow & \downarrow & \searrow^{0} & \\ (\tau_{\geq 1}X)[-1] & \longrightarrow & \tau_{\leq 0}X & \longrightarrow & X & \longrightarrow & \tau_{\geq 1}X \end{array}$$

$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \overset{\simeq}{\to} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof



$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \xrightarrow{\simeq} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof

$$Y \in \mathcal{C}^{\leq 0}$$

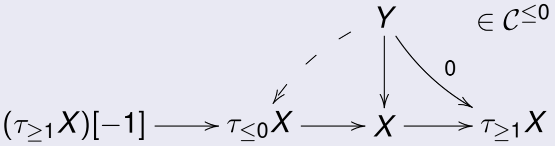
$$\begin{array}{ccccccc} & & & & Y & & \\ & & & \swarrow & \downarrow & \overset{0}{\searrow} & \\ (\tau_{\geq 1}X)[-1] & \longrightarrow & \tau_{\leq 0}X & \longrightarrow & X & \longrightarrow & \tau_{\geq 1}X \end{array}$$

$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \stackrel{\simeq}{\to} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

Idea of proof



$$0 = (Y, \tau_{\geq 1}X[-1]) \to (Y, \tau_{\leq 0}X) \overset{\simeq}{\to} (Y, X) \to (Y, \tau_{\geq 1}X) = 0$$

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

- For $X \in \mathcal{C}$, the following are equivalent:
  (i) $X \in \mathcal{C}^{\leq 0}$;
  (ii) $\tau_{\geq 1} X = 0$;
  (iii) $\tau_{\leq 0} X \to X$ is an isomorphism;
  (iv) $\mathrm{Hom}_{\mathcal{C}}(X, \mathcal{C}^{\geq 1}) = 0$.

- In particular, $\mathcal{C}^{\leq 0}$ is extension closed in $\mathcal{C}$.

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

- For $X \in \mathcal{C}$, the following are equivalent:
  (i) $X \in \mathcal{C}^{\leq 0}$;
  (ii) $\tau_{\geq 1} X = 0$;
  (iii) $\tau_{\leq 0} X \to X$ is an isomorphism;
  (iv) $\mathrm{Hom}_{\mathcal{C}}(X, \mathcal{C}^{\geq 1}) = 0$.

- In particular, $\mathcal{C}^{\leq 0}$ is extension closed in $\mathcal{C}$.

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

- For $X \in \mathcal{C}$, the following are equivalent:
  (i) $X \in \mathcal{C}^{\leq 0}$;
  (ii) $\tau_{\geq 1} X = 0$;
  (iii) $\tau_{\leq 0} X \to X$ is an isomorphism;
  (iv) $\mathrm{Hom}_{\mathcal{C}}(X, \mathcal{C}^{\geq 1}) = 0$.

- In particular, $\mathcal{C}^{\leq 0}$ is extension closed in $\mathcal{C}$.

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

- For $X \in \mathcal{C}$, the following are equivalent:
  (i) $X \in \mathcal{C}^{\leq 0}$;
  (ii) $\tau_{\geq 1} X = 0$;
  (iii) $\tau_{\leq 0} X \to X$ is an isomorphism;
  (iv) $\mathrm{Hom}_{\mathcal{C}}(X, \mathcal{C}^{\geq 1}) = 0$.

- In particular, $\mathcal{C}^{\leq 0}$ is extension closed in $\mathcal{C}$.

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

- For $X \in \mathcal{C}$, the following are equivalent:
  (i) $X \in \mathcal{C}^{\leq 0}$;
  (ii) $\tau_{\geq 1} X = 0$;
  (iii) $\tau_{\leq 0} X \to X$ is an isomorphism;
  (iv) $\mathrm{Hom}_{\mathcal{C}}(X, \mathcal{C}^{\geq 1}) = 0$.

- In particular, $\mathcal{C}^{\leq 0}$ is extension closed in $\mathcal{C}$.

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

- For $X \in \mathcal{C}$, the following are equivalent:
  - (i) $X \in \mathcal{C}^{\leq 0}$;
  - (ii) $\tau_{\geq 1} X = 0$;
  - (iii) $\tau_{\leq 0} X \to X$ is an isomorphism;
  - (iv) $\mathrm{Hom}_{\mathcal{C}}(X, \mathcal{C}^{\geq 1}) = 0$.

- In particular, $\mathcal{C}^{\leq 0}$ is extension closed in $\mathcal{C}$.

Some properties of t-structures

- $\tau_{\leq 0}$ is a functor ; right adjoint to the inclusion $\mathcal{C}^{\leq 0} \subset \mathcal{C}$.
- $\tau_{\geq 1}$ is a functor ; left adjoint to the inclusion $\mathcal{C}^{\geq 1} \subset \mathcal{C}$.

- For $X \in \mathcal{C}$, the following are equivalent:
  (i) $X \in \mathcal{C}^{\leq 0}$;
  (ii) $\tau_{\geq 1}X = 0$;
  (iii) $\tau_{\leq 0}X \to X$ is an isomorphism;
  (iv) $\mathrm{Hom}_{\mathcal{C}}(X, \mathcal{C}^{\geq 1}) = 0$.

- In particular, $\mathcal{C}^{\leq 0}$ is extension closed in $\mathcal{C}$.

Definition of the heart

Let $a \leq b$, then

$$\begin{array}{rcccc} \tau_{\geq a} & : & \mathcal{C}^{\leq b} & \longrightarrow & \mathcal{C}^{\leq b} \\ \tau_{\leq b} & : & \mathcal{C}^{\geq a} & \longrightarrow & \mathcal{C}^{\geq a} \end{array}$$

Define $\mathcal{A} = \mathcal{C}^{\leq 0} \cap \mathcal{C}^{\geq 0}$, called the heart of the t-structure.
For all $X \in \mathcal{C}$, $H^0 X = \tau_{\geq 0}\tau_{\leq 0} X \xrightarrow{\text{can}\simeq} \tau_{\leq 0}\tau_{\geq 0} X \in \mathcal{A}$
Define $H^n X = H^0(X[n])$

Example

$\mathcal{C}$ is the derived category of an abelian category $\mathcal{A}_b$.
Canonical t-structure given by: $X \in \mathcal{C}^{\leq 0}$ iff the cohomology of X vanishes in positive degrees.
Then $\mathcal{A} = \mathcal{A}_b$ and H^n coincides with n-th cohomology.

Definition of the heart

Let $a \leq b$, then

$$\begin{array}{cccc} \tau_{\geq a} & : & \mathcal{C}^{\leq b} & \longrightarrow & \mathcal{C}^{\leq b} \\ \tau_{\leq b} & : & \mathcal{C}^{\geq a} & \longrightarrow & \mathcal{C}^{\geq a} \end{array}$$

Define $\mathcal{A} = \mathcal{C}^{\leq 0} \cap \mathcal{C}^{\geq 0}$, called the heart of the t-structure.
For all $X \in \mathcal{C}$, $H^0 X = \tau_{\geq 0}\tau_{\leq 0} X \overset{\text{can} \simeq}{\longrightarrow} \tau_{\leq 0}\tau_{\geq 0} X \in \mathcal{A}$
Define $H^n X = H^0(X[n])$

Example

$\mathcal{C}$ is the derived category of an abelian category $\mathcal{A}_b$.
Canonical t-structure given by: $X \in \mathcal{C}^{\leq 0}$ iff the cohomology of X vanishes in positive degrees.
Then $\mathcal{A} = \mathcal{A}_b$ and H^n coincides with n-th cohomology.

Definition of the heart

Let $a \leq b$, then

$$\begin{array}{cccc} \tau_{\geq a} & : & \mathcal{C}^{\leq b} & \longrightarrow & \mathcal{C}^{\leq b} \\ \tau_{\leq b} & : & \mathcal{C}^{\geq a} & \longrightarrow & \mathcal{C}^{\geq a} \end{array}$$

Define $\mathcal{A} = \mathcal{C}^{\leq 0} \cap \mathcal{C}^{\geq 0}$, called the heart of the t-structure.
For all $X \in \mathcal{C}$, $H^0 X = \tau_{\geq 0}\tau_{\leq 0} X \overset{\text{can}\simeq}{\longrightarrow} \tau_{\leq 0}\tau_{\geq 0} X \in \mathcal{A}$
Define $H^n X = H^0(X[n])$

Example

$\mathcal{C}$ is the derived category of an abelian category $\mathcal{A}_b$.
Canonical t-structure given by: $X \in \mathcal{C}^{\leq 0}$ iff the cohomology of X vanishes in positive degrees.
Then $\mathcal{A} = \mathcal{A}_b$ and H^n coincides with n-th cohomology.

Definition of the heart

Let $a \leq b$, then

$$\begin{array}{cccc} \tau_{\geq a} & : & \mathcal{C}^{\leq b} & \longrightarrow & \mathcal{C}^{\leq b} \\ \tau_{\leq b} & : & \mathcal{C}^{\geq a} & \longrightarrow & \mathcal{C}^{\geq a} \end{array}$$

Define $\mathcal{A} = \mathcal{C}^{\leq 0} \cap \mathcal{C}^{\geq 0}$, called the heart of the t-structure.

For all $X \in \mathcal{C}$, $H^0 X = \tau_{\geq 0}\tau_{\leq 0} X \xrightarrow{\text{can}\simeq} \tau_{\leq 0}\tau_{\geq 0} X \in \mathcal{A}$

Define $H^n X = H^0(X[n])$

Example

$\mathcal{C}$ is the derived category of an abelian category $\mathcal{A}_b$.

Canonical t-structure given by: $X \in \mathcal{C}^{\leq 0}$ iff the cohomology of X vanishes in positive degrees.

Then $\mathcal{A} = \mathcal{A}_b$ and H^n coincides with n-th cohomology.

Definition of the heart

Let $a \leq b$, then

$$\begin{array}{rcccc} \tau_{\geq a} & : & \mathcal{C}^{\leq b} & \longrightarrow & \mathcal{C}^{\leq b} \\ \tau_{\leq b} & : & \mathcal{C}^{\geq a} & \longrightarrow & \mathcal{C}^{\geq a} \end{array}$$

Define $\mathcal{A} = \mathcal{C}^{\leq 0} \cap \mathcal{C}^{\geq 0}$, called the heart of the t-structure.
For all $X \in \mathcal{C}$, $H^0 X = \tau_{\geq 0}\tau_{\leq 0} X \xrightarrow{\text{can}\simeq} \tau_{\leq 0}\tau_{\geq 0} X \in \mathcal{A}$
Define $H^n X = H^0(X[n])$

Example

$\mathcal{C}$ is the derived category of an abelian category $\mathcal{A}_b$.
Canonical t-structure given by: $X \in \mathcal{C}^{\leq 0}$ iff the cohomology of X vanishes in positive degrees.
Then $\mathcal{A} = \mathcal{A}_b$ and H^n coincides with n-th cohomology.

Definition of the heart

Let $a \leq b$, then

$$\begin{array}{rcccl} \tau_{\geq a} & : & \mathcal{C}^{\leq b} & \longrightarrow & \mathcal{C}^{\leq b} \\ \tau_{\leq b} & : & \mathcal{C}^{\geq a} & \longrightarrow & \mathcal{C}^{\geq a} \end{array}$$

Define $\mathcal{A} = \mathcal{C}^{\leq 0} \cap \mathcal{C}^{\geq 0}$, called the heart of the t-structure.
For all $X \in \mathcal{C}$, $H^0 X = \tau_{\geq 0}\tau_{\leq 0} X \xrightarrow{\text{can}\simeq} \tau_{\leq 0}\tau_{\geq 0} X \in \mathcal{A}$
Define $H^n X = H^0(X[n])$

Example

$\mathcal{C}$ is the derived category of an abelian category $\mathcal{A}_b$.
Canonical t-structure given by: $X \in \mathcal{C}^{\leq 0}$ iff the cohomology of X vanishes in positive degrees.
Then $\mathcal{A} = \mathcal{A}_b$ and H^n coincides with n-th cohomology.

Definition of the heart

Let $a \leq b$, then

$$\begin{array}{cccc} \tau_{\geq a} & : & \mathcal{C}^{\leq b} & \longrightarrow & \mathcal{C}^{\leq b} \\ \tau_{\leq b} & : & \mathcal{C}^{\geq a} & \longrightarrow & \mathcal{C}^{\geq a} \end{array}$$

Define $\mathcal{A} = \mathcal{C}^{\leq 0} \cap \mathcal{C}^{\geq 0}$, called the heart of the t-structure.
For all $X \in \mathcal{C}$, $H^0X = \tau_{\geq 0}\tau_{\leq 0}X \xrightarrow{\text{can}\simeq} \tau_{\leq 0}\tau_{\geq 0}X \in \mathcal{A}$
Define $H^nX = H^0(X[n])$

Example

$\mathcal{C}$ is the derived category of an abelian category $\mathcal{A}_b$.
Canonical t-structure given by: $X \in \mathcal{C}^{\leq 0}$ iff the cohomology of X vanishes in positive degrees.
Then $\mathcal{A} = \mathcal{A}_b$ and H^n coincides with n-th cohomology.

Definition of the heart

Let $a \leq b$, then

$$\begin{array}{cccc} \tau_{\geq a} & : & \mathcal{C}^{\leq b} & \longrightarrow & \mathcal{C}^{\leq b} \\ \tau_{\leq b} & : & \mathcal{C}^{\geq a} & \longrightarrow & \mathcal{C}^{\geq a} \end{array}$$

Define $\mathcal{A} = \mathcal{C}^{\leq 0} \cap \mathcal{C}^{\geq 0}$, called the heart of the t-structure.
For all $X \in \mathcal{C}$, $H^0 X = \tau_{\geq 0}\tau_{\leq 0} X \xrightarrow{\text{can}\simeq} \tau_{\leq 0}\tau_{\geq 0} X \in \mathcal{A}$
Define $H^n X = H^0(X[n])$

Example

$\mathcal{C}$ is the derived category of an abelian category $\mathcal{A}_b$.
Canonical t-structure given by: $X \in \mathcal{C}^{\leq 0}$ iff the cohomology of X vanishes in positive degrees.
Then $\mathcal{A} = \mathcal{A}_b$ and H^n coincides with n-th cohomology.

Theorem (Beilinson–Bernstein–Deligne):

Theorem

(i) *The heart $\mathcal{A}$ is an abelian category.*

(ii) $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ *is a short exact sequence in $\mathcal{A}$ if and only if there exists ε such that* $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{\varepsilon} A[1]$ *is a triangle in $\mathcal{C}$.*

(iii) $H^0 : \mathcal{C} \longrightarrow \mathcal{A}$ *is a cohomological functor: Any triangle* $X \to Y \to Z \to X[1]$ *in $\mathcal{C}$ induces a long exact sequence*

$$\cdots \to H^{-1}Z \to H^0X \to H^0Y \to H^0Z \to H^1X \to \cdots$$

in $\mathcal{A}$.

Theorem (Beilinson–Bernstein–Deligne):

Theorem

(i) *The heart $\mathcal{A}$ is an abelian category.*

(ii) $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ *is a short exact sequence in* $\mathcal{A}$ *if and only if there exists* ε *such that* $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{\varepsilon} A[1]$ *is a triangle in* $\mathcal{C}$.

(iii) $H^0 : \mathcal{C} \longrightarrow \mathcal{A}$ *is a cohomological functor: Any triangle* $X \to Y \to Z \to X[1]$ *in* $\mathcal{C}$ *induces a long exact sequence*

$$\cdots \to H^{-1}Z \to H^0X \to H^0Y \to H^0Z \to H^1X \to \cdots$$

in $\mathcal{A}$.

Theorem (Beilinson–Bernstein–Deligne):

Theorem

(i) *The heart $\mathcal{A}$ is an abelian category.*

(ii) $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ *is a short exact sequence in* $\mathcal{A}$ *if and only if there exists* ε *such that* $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{\varepsilon} A[1]$ *is a triangle in* $\mathcal{C}$.

(iii) $H^0 : \mathcal{C} \longrightarrow \mathcal{A}$ *is a cohomological functor: Any triangle* $X \to Y \to Z \to X[1]$ *in* $\mathcal{C}$ *induces a long exact sequence*

$$\cdots \to H^{-1}Z \to H^0X \to H^0Y \to H^0Z \to H^1X \to \cdots$$

in $\mathcal{A}$.

Theorem (Beilinson–Bernstein–Deligne):

Theorem

(i) *The heart $\mathcal{A}$ is an abelian category.*

(ii) $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ *is a short exact sequence in $\mathcal{A}$ if and only if there exists ε such that* $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{\varepsilon} A[1]$ *is a triangle in $\mathcal{C}$.*

(iii) $H^0 : \mathcal{C} \longrightarrow \mathcal{A}$ *is a cohomological functor: Any triangle* $X \to Y \to Z \to X[1]$ *in $\mathcal{C}$ induces a long exact sequence*

$$\cdots \to H^{-1}Z \to H^0X \to H^0Y \to H^0Z \to H^1X \to \cdots$$

in $\mathcal{A}$.

Theorem (Beilinson–Bernstein–Deligne):

Theorem

(i) *The heart $\mathcal{A}$ is an abelian category.*

(ii) $0 \to A \xrightarrow{f} B \xrightarrow{g} C \to 0$ *is a short exact sequence in $\mathcal{A}$ if and only if there exists ε such that* $A \xrightarrow{f} B \xrightarrow{g} C \xrightarrow{\varepsilon} A[1]$ *is a triangle in $\mathcal{C}$.*

(iii) $H^0 : \mathcal{C} \longrightarrow \mathcal{A}$ *is a cohomological functor: Any triangle* $X \to Y \to Z \to X[1]$ *in $\mathcal{C}$ induces a long exact sequence*

$$\cdots \to H^{-1}Z \to H^0X \to H^0Y \to H^0Z \to H^1X \to \cdots$$

in $\mathcal{A}$.

Sketch of proof:

$(\tau_{\leq -1} Z)[-1] \in \mathcal{A}$ $\tau_{\leq -1} Z$

$Z[-1]$ A B Z

$\tau_{\geq 0} Z \in \mathcal{A}$

- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:

$(\tau_{\leq -1}Z)[-1] \in \mathcal{A}$ $\qquad$ $\tau_{\leq -1}Z$

$Z[-1]$ $\qquad$ A $\qquad$ B $\qquad$ Z

$\tau_{\geq 0}Z \in \mathcal{A}$

- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:

$(\tau_{\leq -1} Z)[-1] \in \mathcal{A}$ $\qquad$ $\tau_{\leq -1} Z$

$Z[-1]$ $\qquad$ A $\qquad$ B $\qquad$ Z

$\tau_{\geq 0} Z \in \mathcal{A}$

- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:

$$(\tau_{\leq -1}Z)[-1] \in \mathcal{A} \qquad \tau_{\leq -1}Z$$

$$Z[-1] \qquad A \xrightarrow{f} B \qquad Z$$

$$\tau_{\geq 0}Z \in \mathcal{A}$$

- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:

$(\tau_{\leq -1} Z)[-1] \in \mathcal{A}$ $\qquad$ $\tau_{\leq -1} Z$

$$Z[-1] \longrightarrow A \xrightarrow{f} B \longrightarrow Z$$

$\tau_{\geq 0} Z \in \mathcal{A}$

- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:

$$
\begin{array}{ccccccc}
(\tau_{\leq -1} Z)[-1] \in \mathcal{A} & & & & & & \tau_{\leq -1} Z \\
 & & & & & & \downarrow \\
Z[-1] & \longrightarrow & A & \xrightarrow{f} & B & \longrightarrow & Z \\
 & & & & & & \downarrow \\
 & & & & & & \tau_{\geq 0} Z \in \mathcal{A}
\end{array}
$$

- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:

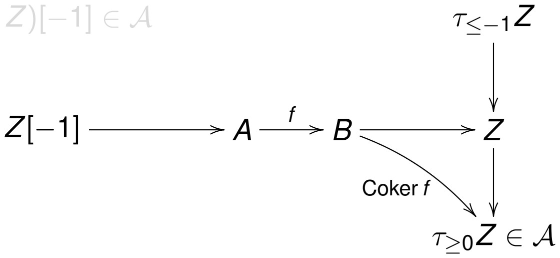
$(\tau_{\leq -1}Z)[-1] \in \mathcal{A}$

$$\begin{array}{ccccccc} & & & & & & \tau_{\leq -1}Z \\ & & & & & & \downarrow \\ Z[-1] & \longrightarrow & A & \xrightarrow{f} & B & \longrightarrow & Z \\ & & & & & & \downarrow \\ & & & & & & \tau_{\geq 0}Z \in \mathcal{A} \end{array}$$

- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

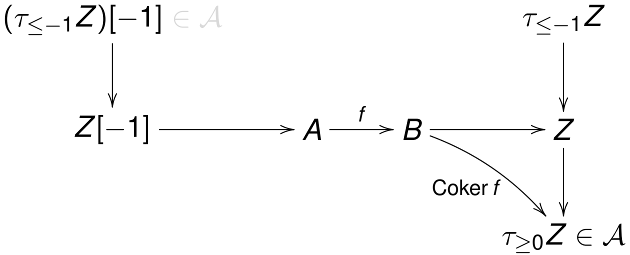
Sketch of proof:

$(\tau_{\leq -1}Z)[-1] \in \mathcal{A}$



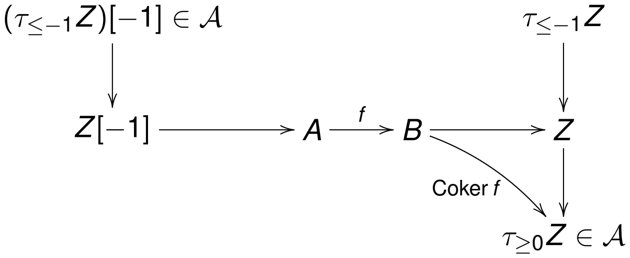
- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:



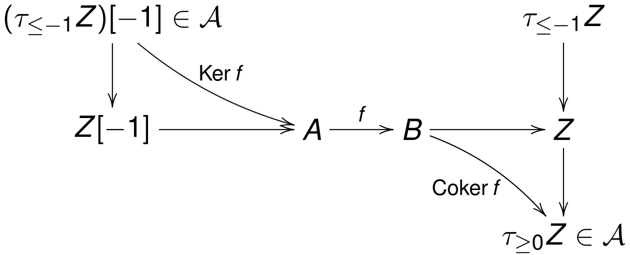
- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:



- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Sketch of proof:



- $\mathcal{C}^{\leq 0}$ is extension closed
- $\mathcal{C}(\mathcal{A}, \mathcal{A}[-1]) = 0$

Non-degenerate t-structures

A t-structure is non-degenerate if

$$\bigcap_n \mathcal{C}^{\leq n} = 0 = \bigcap_n \mathcal{C}^{\geq n}$$

Proposition:

Let the t-structure be non-degenerate. Then

$$X \in \mathcal{C}^{\leq 0} \iff H^i X = 0 \ \forall i > 0$$

$$X \in \mathcal{C}^{\geq 0} \iff H^i X = 0 \ \forall i < 0$$

Non-degenerate t-structures

A t-structure is non-degenerate if

$$\bigcap_n \mathcal{C}^{\leq n} = 0 = \bigcap_n \mathcal{C}^{\geq n}$$

Proposition:

Let the t-structure be non-degenerate. Then

$$X \in \mathcal{C}^{\leq 0} \iff H^i X = 0 \ \ \forall i > 0$$

$$X \in \mathcal{C}^{\geq 0} \iff H^i X = 0 \ \ \forall i < 0$$

Non-degenerate t-structures

A t-structure is non-degenerate if

$$\bigcap_n \mathcal{C}^{\leq n} = 0 = \bigcap_n \mathcal{C}^{\geq n}$$

Proposition:

Let the t-structure be non-degenerate. Then

$$X \in \mathcal{C}^{\leq 0} \iff H^i X = 0 \ \forall i > 0$$

$$X \in \mathcal{C}^{\geq 0} \iff H^i X = 0 \ \forall i < 0$$

Non-degenerate t-structures

A t-structure is non-degenerate if

$$\bigcap_n \mathcal{C}^{\leq n} = 0 = \bigcap_n \mathcal{C}^{\geq n}$$

Proposition:

Let the t-structure be non-degenerate. Then

$$X \in \mathcal{C}^{\leq 0} \iff H^i X = 0 \ \forall i > 0$$

$$X \in \mathcal{C}^{\geq 0} \iff H^i X = 0 \ \forall i < 0$$

Non-degenerate t-structures

A t-structure is non-degenerate if

$$\bigcap_n \mathcal{C}^{\leq n} = 0 = \bigcap_n \mathcal{C}^{\geq n}$$

Proposition:

Let the t-structure be non-degenerate. Then

$$X \in \mathcal{C}^{\leq 0} \iff H^i X = 0 \ \ \forall i > 0$$

$$X \in \mathcal{C}^{\geq 0} \iff H^i X = 0 \ \ \forall i < 0$$