WORKING SEMINAR "SHEAVES IN REPRESENTATION THEORY"

The participants are invited to choose a talk in the following list. We have classified the talks into several streams, which we have abbreviated as follows:

C	Combinatorics,
RT	Representation theory,
GT	Geometry and topology,
HA	Homological algebra,
PS	Perverse sheaves and parity sheaves,
R	Invited research talks.

If you are interested in giving a talk, then

- please choose a talk title that suites you,
- try to give also 1 or 2 alternatives, and
- write to Geordie Williamson `williamsong@maths.ox.ac.uk`
- Deadline: 28 February 2010.

After the deadline, we will distribute the talks and direct you to the appropriate organiser (whose name is bracketed after the talk title). This organiser will then assist you with further information and references and can suggest people that you can talk to at or near your institution. The organisers are also available to answer any questions that you may have during preparation.

The e-mail addresses of the organisers are as follows:

Peter Fiebig	fiebig@mi.uni-erlangen.de,
Daniel Juteau	daniel.juteau@math.unicaen.fr,
Geordie Williamson	williamsong@maths.ox.ac.uk.

Monday

Morning

C1 *The combinatorics of Weyl groups and Kazhdan-Lusztig polynomials (Fiebig)*
Root systems, Weyl groups, Hecke algebras, Kazhdan–Lusztig polynomials and their inductive calculation [Hum90, Bou02].

RT1 *An introduction to category $\mathcal{O}$ (Fiebig)*
Category $\mathcal{O}$ (for finite dimensional complex semisimple Lie algebras $\mathfrak{g}$), simple highest weight modules, characters, linkage, block decomposition, BGG-reciprocity, the Kazhdan–Lusztig conjecture [Hum08].

C2 *Moment graphs (Fiebig)*
Definition, sheaves on moment graphs, Braden–MacPherson sheaves, GKM-graphs, the multiplicity conjecture [Fie09, BM01].

Afternoon

O1 *Overview*[1]
An approach to the Kazhdan-Lusztig conjecture using moment graphs and the topology of Schubert varieties. Deformed category $\mathcal{O}$, the equivariant derived category, parity sheaves, the decomposition theorem.

GT1 *The geometry of Schubert varieties (Williamson)*
Flag varieties, partial flag varieties, Schubert varieties, the Bott-Samelson resolution, the moment graph of a Schubert variety [Bri03].

GT2 *Introduction to algebraic topology (Juteau)*
Simplicial (co)homology and a sketch of variants (e.g. for CW complexes), long exact sequences, examples [Hat02].

E1 *First exercise class*
Calculations of Kazhdan-Lusztig polynomials, examples of smooth and singular Schubert varieties, calculations of moment graphs, cohomology calculations.

[1]This talk will be given by an organiser.

Tuesday

Morning

RT2 *Deformed category $\mathcal{O}$ (Fiebig)*
Deformations of standard and projective modules, (with the example of $\mathfrak{sl}_2$-case treated in detail), Verma flags, the functor $\mathbb{V}$ to sheaves on moment graphs [Fie08].

RT3 *Deformed category $\mathcal{O}$ and moment graphs (Fiebig)*
The theorem that $\mathbb{V}$ of a deformed projective object is a Braden-MacPherson sheaf. The multiplicity conjecture implies the Kazhdan–Lusztig conjecture [Fie08].

HA1 *Derived and triangulated categories (Juteau)*
Exactness properties, triangulated categories, derived categories, derived functors, the relation between shifted Hom and Ext [Wei94].

Afternoon

GT3 *Sheaves and their derived category (Juteau)*
Sheaves, direct and inverse images, derived category of sheaves, sheaf cohomology, local systems, constructible sheaves, stability under direct and inverse images.

GT4 *The Grothendieck formalism and Verdier duality (Juteau)*
Direct image with compact support, exceptional inverse image, duality [Ive86]. Examples: inclusion of a smooth closed subvariety in a smooth variety (Gysin), the case of a fibration.

E2 *Second exercise class*
Deformed category $\mathcal{O}$, examples of constructible sheaves and complexes, calculation with the six operations.

Wednesday

Morning

GT5 *Equivariant cohomology and the equivariant derived category (Williamson)*
Classifying spaces, equivariant cohomology, examples, the equivariant derived category, Grothendieck formalism and the forgetful functor [BL94].

PS1 *t-structures and recollement (Juteau)*
Definition, the heart is an abelian category, recollement of *t*-structures, description of simple objects in a recollement situation [BBD82].

PS2 *Perverse sheaves (Juteau)*
Definition, simple objects, statement of the decomposition theorem [BBD82].

Afternoon

R1-2 *Three research talks*

E3 *Third exercise class*
Examples of *t*-structures, examples of intersection cohomology complexes, carrying out the Deligne construction, calculating stalks using the decomposition theorem.

THURSDAY

Morning

PS3 *Stalks of IC complexes on the flag variety (Williamson)*
Proof of the theorem of Kazhdan and Lusztig that the dimension of the local cohomology of intersection cohomology complexes is given by Kazhdan-Lusztig polynomials [Spr82, Soe00].

PS4 *Parity sheaves (Williamson)*
First properties, construction via even resolutions, parity sheaves in characteristic 0, examples [Soe00, JMW09].

PS5 *Parity sheaves and moment graphs (Williamson)*
The functor $\mathbb{W}$ from parity sheaves to sheaves on the moment graphs. The theorem relating parity sheaves and Braden-MacPherson sheaves [FW]. The last step in the proof of the Kazhdan-Lusztig conjecture (assuming the decomposition theorem).

Afternoon

R3-4-5 *Three research talks*

E4 *Fourth exercise class*
Examples of parity sheaves and even resolutions. Examples of failure of the decomposition theorem. Calculations of stalks of parity sheaves via moment graphs.

FRIDAY

Morning

R6-7-8 *Three research talks*

The list of speakers for the research talks will include:

Braden
Finkelberg
Gaitsgory
Mautner
McGerty
Mirkovic
Rumynin
Stroppel

References

[BBD82] A. A. Beĭlinson, J. Bernstein, and P. Deligne. Faisceaux pervers. In *Analysis and topology on singular spaces, I (Luminy, 1981)*, volume 100 of *Astérisque*, pages 5–171. Soc. Math. France, Paris, 1982.

[BL94] J. Bernstein and V. Lunts. *Equivariant sheaves and functors*, volume 1578 of *Lecture Notes in Mathematics*. Springer-Verlag, Berlin, 1994.

[BM01] T. Braden and R. MacPherson. From moment graphs to intersection cohomology. *Math. Ann.*, 321(3):533–551, 2001.

[Bou02] N. Bourbaki. *Lie groups and Lie algebras. Chapters 4–6*. Elements of Mathematics (Berlin). Springer-Verlag, Berlin, 2002. Translated from the 1968 French original by Andrew Pressley.

[Bri03] M. Brion. Lectures on the geometry of flag varieties. Notes of the summer school "Schubert varieties" in Warsaw, 59 pages, `http://www-fourier.ujf-grenoble.fr/~mbrion/notes.html`, 2003.

[Fie08] P. Fiebig. Sheaves on moment graphs and a localization of Verma flags. *Adv. Math.*, 217(2):683–712, 2008.

[Fie09] P. Fiebig. Moment graphs in representation theory and topology, 2009. Lecture notes, `http://www.mi.uni-erlangen.de/~fiebig/Skript_Cologne.pdf`.

[FW] P. Fiebig and G. Williamson. On the p-smooth locus of schubert varieties. In preparation.

[Hat02] A. Hatcher. *Algebraic topology*. Cambridge University Press, Cambridge, 2002.

[Hum90] J. E. Humphreys. *Reflection groups and Coxeter groups*, volume 29 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1990.

[Hum08] J. E. Humphreys. *Representations of semisimple Lie algebras in the BGG category $\mathcal{O}$*, volume 94 of *Graduate Studies in Mathematics*. American Mathematical Society, Providence, RI, 2008.

[Ive86] B. Iversen. *Cohomology of sheaves*. Universitext. Springer-Verlag, Berlin, 1986.

[JMW09] D. Juteau, C. Mautner, and G. Williamson. Parity sheaves. Preprint `arXiv:0906.2994v1`, 2009.

[Soe00] W. Soergel. On the relation between intersection cohomology and representation theory in positive characteristic. *J. Pure Appl. Algebra*, 152(1-3):311–335, 2000. Commutative algebra, homological algebra and representation theory (Catania/Genoa/Rome, 1998).

[Spr82] T. A. Springer. Quelques applications de la cohomologie d'intersection. In *Bourbaki Seminar, Vol. 1981/1982*, volume 92 of *Astérisque*, pages 249–273. Soc. Math. France, Paris, 1982.

[Wei94] C. A. Weibel. *An introduction to homological algebra*, volume 38 of *Cambridge Studies in Advanced Mathematics*. Cambridge University Press, Cambridge, 1994.