Representation Theory Across The Channel

An EPSRC Research Network Proposal – Case for Support

Meinolf Geck, Aberdeen, June 2007

Part I: Previous Research Track Record

The Principal Investigator (PI)

Since September 2005, Principal Investigator Meinolf Geck has been a 6th Century Professor in Pure Mathematics at the University of Aberdeen. Prior to this, he worked and lived for 10 years in France, first as "Chargé de Recherche" (a permanent CNRS research position) at the Université Paris 7 from 1995 to 1999 and then, from 1999 to 2005, as a Full Professor at the University Claude–Bernard Lyon 1. He obtained his PhD (1990) and Habilitation (1994) at the RWTH Aachen (Germany).

The PI has written about 60 research articles and 2 monographs in the area of "Representation Theory", more specifically on representations of finite groups of Lie type (ordinary and modular) and the theory of Hecke algebras. He is also one of the developers of the computer algebra system CHEVIE, which allows symbolic calculations with the above structures.

A guiding principle for his work is the major open problem of determining the irreducible representations in the modular (i.e., non-semisimple) situation. Significant advances are achieved in [11], [8], [9]. Further recent work of the PI concerns Lusztig's conjectures on cells associated to Hecke algebras with unequal parameters ([3], [6], [10], [7]) and rationality properties of unipotent representations of finite groups of Lie type ([2], [4], [5]).

The PI has been actively involved in the organisation of research in "Representation Theory". Jointly with D. Testerman and J. Thévenaz, he organised a 6-month long research programme on "Group Representation Theory" [12] at the Bernoulli Center of the EPFL Lausanne (Switzerland). Jointly with A. Kleshchev and G. Röhrle, he will organise a 6-month long research programme on "Algebraic Lie Theory" at the Isaac Newton Institute in Cambridge (UK).

The Co-Investigator (CoI)

Co-Investigator Iain Gordon has been The Professor of Mathematics at the University of Edinburgh since 2006, having worked at the University of Glasgow since 2000. After graduating in 1998, he has had a variety of visiting positions in Edinburgh, Bielefeld (Germany), Antwerp (Belgium), MSRI (USA), University of Washington (USA) and the University of California at Santa Barbara (USA).

The CoI is the author of around twenty research articles. These papers are in representation theory, and more specifically in the interactions between it, noncommutative algebra, algebraic geometry and combinatorics. His most recent work is in the representation theory of Cherednik algebras, developing connections with algebraic combinatorics, the geometry of Hilbert schemes of points in the plane, Hecke algebras and

noncommutative algebraic geometry, [14], [15], [13]. In 2005 he was awarded the London Mathematical Society's Berwick Prize in recognition of his work.

The CoI has been involved in much UK collaborative activity. He was a founder and organiser of ARTIN (Algebra and Representation Theory in the North), a London Mathematical Society Scheme 3 funded programme which is now run out of Aberdeen by Alexey Sevastyanov. The CoI was the organiser of a European Science Foundation training workshop on Derived Categories in 2003, and will be a co-organiser of forthcoming meetings on "Cherednik Algebras" (2007) at the ICMS, "Frontiers of Science" (2008) in Berlin in collaboration with the Royal Society and the Junge Akademie, and "Categorification and Geometrization", an Isaac Newton Institute sponsored event (2009). He is currently a member of the Council of the London Mathematical Society. The CoI is involved in an application for an FP7 Training Network grant in Representation Theory: this currently features the University of Edinburgh as a node (together with nodes in seven other countries). We would like the UK network to become the official node if this application is successful.

Network Partners

The Network will involve research teams from all major UK Mathematics Departments, including Aberdeen, Bath, Birmingham, Bristol, Cambridge, Edinburgh, Exeter, Glasgow, Kent, Leeds, Leicester, Liverpool, London City College, Imperial College London, King's College London, Queen Mary University of London, Loughborough, East Anglia, Nottingham, Oxford, Southampton, Sheffield, Swansea, Warwick, York.

At this stage, a mailing list has been set-up, with about 140 names, and information related to this proposal has been widely circulated. The feedback has been uniformly enthusiastic.

Our list includes leading researchers from a wide spectrum of topics related to representation theory and its connections to neighbouring areas such as algebraic topology, geometry, number theory, computer science and mathematical physics. The steering committee which will be set-up in connection to this network includes representatives of major directions in representation theory. We will begin with a relative small committee, including

- PI (Aberdeen; Representations of finite groups, Hecke algebras, computational aspects, . . .),

- CoI (Edinburgh; noncommutative algebra, algebraic geometry, combinatorics, . . .),

- Joseph Chuang (Bristol; categorification, connections to mathematical physics, . . .),

- John Greenlees (Sheffield; algebraic topology, homological algebra, . . .),

- Raphael Rouquier (Oxford; block theory, Broué's conjectures, geometric representation theory, . . .),

- Shaun Stevens (East Anglia; automorphic forms, p-adic groups, number theory, . . .).

The composition of the committee will be revised every year.

As well as setting up this UK network we plan to "twin" the Network with two existing networks in France:

- "GDR 2432: Algèbre Non-Commutative et Théorie des Invariants en Théorie des Represéntations" (contact: Prof. B. Leclerc, Université de Caen);

- "GDR 2249: Groupes, Géometrie et Représentations" (contact: Prof. B. Rémy, Université Lyon 1).

Part 2: The Proposed Network

Introduction

In 2006, through the mediation of the British Embassy in Paris, EPSRC (for the UK side) and CNRS (for the French side) discussed the possibility of setting up a bilateral collaboration for the mutual benefit of mathematicians in both countries, especially for supporting and encouraging mobility of early career researchers. The suggestion was to create a UK-based research network which would then be twinned with already existing networks in France.

"Representation Theory" was identified as an area that could benefit from a bilateral collaboration of this type, where the French networks are "GDR 2432: Algèbre Non-Commutative et Théorie des Invariants en Théorie des Represéntations" (contact: Prof. B. Leclerc, Université de Caen) and "GDR 2249: Groupes, Géometrie et Représentations" (contact: Prof. B. Rémy, Université Lyon 1).

After a preliminary round of consultations, both partners agreed to proceed with the planning. The recently established research group in representation theory at the University of Aberdeen is one of the strongest in the country, covering a wide range of topics in representation theory. It was decided that, as far as the UK is concerned, the administrative side of the network would be coordinated by the Department of Mathematical Sciences at the University of Aberdeen, with Meinolf Geck as PI and Iain Gordon (Edinburgh) as CoI. Whence the present proposal.

Scientific Background

Representation theory is one of the most vibrant fields of mathematics today. Its history, stretching back to the pioneering work of Frobenius, Burnside and Schur, is a story rich in innovation, implementation of techniques from throughout mathematics, and application to all the sciences. Representation theory is visible on the general mathematical scene through large-scale, semester or even year long research programmes with a distinctive inter-disciplinary flavour, e.g., MSRI (Berkeley, USA, 1988, 1990, 2008), Isaac Newton Institute (Cambridge, UK, 1997, 2009), Bernoulli Center (Lausanne, Switzerland, 2005), Institute for Advanced Study (Princeton, USA, 2007–2008). More spectacularly, recent Fields Medals have been awarded to researchers in representation theory: Lafforgue (2002), Okounkov (2006).

The UK is a major international player in this field with leading practitioners throughout the country.

Some History.

Initially, representation theory was concerned with the study of properties of abstract groups via their representations as linear transformations of vector spaces. This idea was soon extended to include other mathematical structures, such as associative algebras, and Lie and Hopf algebras. In this broad sense, representation theory provides basic tools and methods for studying "symmetries" arising in a huge variety of situations, ranging from classical geometry through computer science and on to natural phenomena in physics and chemistry.

The original theory was mainly concerned with representations over the fields of real or complex numbers. New phenomena and difficulties, as well as new connections with other areas (like number theory), arise through the study of representations over p-adic fields or fields of positive characteristic. The introduction of geometric methods, for example the linearisation of group actions by means of cohomology theories, has revolutionised the field. It led to a flow of new ideas and results between several disciplines. The theoretical development has been driven by an abundance of challenging basic problems, like finding character formulae for irreducible representations. Major open conjectures have led to vast research programmes, the most prominent of which probably is the Langlands Programme. Other examples are the conjectures of Alperin, Broué and Dade in the modular representation theory of finite groups, Benson's regularity conjecture about

the characteristic p cohomology of finite groups, Lusztig's conjectures on the irreducible representations of simple Lie algebras in positive characteristic.

In dealing with these problems, highly complex chains of theoretical arguments ranging through many fields of modern mathematics can go hand in hand with large-scale computer-aided calculations. Nowadays, representation theory is a rich mathematical discipline of its own, deeply connected to a variety of areas such as group theory, ring theory, number theory, geometry, algebraic topology, computational algebra, integrable systems, category theory, combinatorics, and mathematical physics.

During the last decade, from different points of view and with a variety of potential applications, new structures have arisen: various types of q-Schur algebras, cyclotomic Hecke algebras, finite W-algebras, double affine Hecke algebras, cluster algebras, p-local finite groups and fusion systems. The cyclotomic algebras, for example, arose from Broué's abelian defect group conjecture (a conjecture about representations of finite groups in positive characteristic); this is a typical example of cross-fertilisation among several disciplines. Many of these new structures have interpretations and incarnations in different disciplines (algebra, geometry, mathematical physics). A researcher working in one of these areas is not necessarily familiar (or even primarily concerned) with all the subtleties and properties in the other.

The Need for Interdisciplinary Interaction.

Given this background, it is essential to provide researchers (of all ages!) with opportunities for interdisciplinary interaction, the chance to develop a broader vision of the fundamental underpinnings and future directions in representation theory, and to learn about neighbouring disciplines. According to the INTERNATIONAL REVIEW OF UK RESEARCH IN MATHEMATICS (2004) [1], "seminars, meetings and workshops, as well as short-term informal visits, are a key source of information about what is happening at the international level, despite the growing importance of electronic communications. Initiatives which put researchers from different centers in contact on a regular basis [...] should be helped and encouraged."

BLOC (Bristol Leicester Oxford - Colloquium) and ARTIN (Algebra and Representation Theory in the North) are two examples of successful programmes for regional and interregional cooperation in the UK. However, they are rather limited in scope and mainly facilitate good relations among the representation theory community. The proposed network is much more ambitious, truly national in scale, interdisciplinary in nature, with an added international component; it is expected to have a significant positive impact on the quality of research in the UK.

The UK/France Aspect.

Both the UK and France are playing a leading role in various aspects of representation theory. According to the INTERNATIONAL REVIEW OF UK RESEARCH IN MATHEMATICS [1], "in algebra, world-class work is being done in infinite group theory, finite groups, noncommutative algebra and representation theory. All these areas lean on an old and strong tradition, and benefit from the presence in the UK of some of the best specialists who are active, inventive, and attract young mathematicians from all over the world." Indeed, there are a lot of people working in representation theory across the whole country, but they are spread from north to south. We have the capacity, and the will, for more interaction and more PhD students, postdocs and visitors. Representation theory is a very broad, fast-moving and interdisciplinary area and so we need to have regular high-level meetings to keep up with developments. There is a lot of demand for this, see, e.g., the LMS Invited Lectures on the "Geometric Langlands Correspondence" at Oxford (2007), or the ad-hoc meetings "Representation Theory of p-adic groups" at King's College London (2007) and on "Cherednik algebras" at the ICMS Edinburgh (2007): it is essential for such activities to be delivered as a matter of course, not chance.

Mathematicians in France have high-profile expertise in some fields where the UK is under-represented: for instance in combinatorial representation theory, the modern theory of automorphic forms, the Langlands

programme; the French are world leaders in the representation theory of p-adic reductive groups. They complement areas where we have people working, but not yet too many, e.g., geometric representation theory. We have common areas of strength: Lie theory, ring theory, group representation theory. Our UK researchers will greatly benefit from visiting mathematical groups in France, and so then will our mathematics. Links exist, but again they are ad-hoc, e.g., the Scottish Algebra Day has invited French speakers, the French "Solstice" conferences always have UK speakers.

Program and Methodology

Fundamental Aims and Objectives of the Network

Representation theory is one of the strong areas of research in the UK, with some activities already taking place (including invited lecture series, meetings, regional and interregional cooperations, ...). Characteristic features of the developments over the last few decades are

- the new level of complexity and diversity of the theories and methods involved (e.g., geometrisation, categorification, ...);

- the rapidly developing connections with other areas such as algebraic topology, homological algebra, algebraic geometry, number theory and theoretical physics.

The ad-hoc organisation of meetings, regional and interregional cooperations like BLOC and ARTIN are not sufficient to keep up with this, especially regarding the training of early career researchers and the recruitment of PhD students and post-docs. Thus, the most fundamental aims of this proposal are:

- To provide a common organised framework for inter-disciplinary activities in representation theory (and its applications) across the UK.

- To provide early career researchers in representation theory with opportunities to broaden their knowledge, learn about cutting-edge developments, meet neighbouring disciplines: all essential for lift-off to a successful research career.

- To act as a basis for international cooperation, more precisely:

    - by "twinning" with two French networks (through a formal agreement between EPSRC and CNRS);
    - as official node for an FP7 Training Network grant in Representation Theory (the application for which is currently under way, with the CoI being involved).

In order to meet these aims, we need to encourage and support initiatives putting researchers from different centres and with different backgrounds in contact on a regular basis. Thus, the objectives of the network are:

- To organise, in coordination with the French side, annual workshops or high-level instructional conferences on topics of current interest in representation theory.

- To facilitate the timely organisation of short meetings and invited lecture series, in response to recent developments in representation theory.

- To encourage and support mobility of young researchers through a short-visit scheme.

- To focus and promote international cooperation in representation theory, especially with France.

The Proposed Three-Year Programme (2008–2010)

The network will support and organise the following activities.

● **Short Visit Scheme.**

All network partners are invited to contact members of the steering committee concerning such visits. As a general rule, the network provides support for subsistence for visitors to the UK and travel support for visits out of UK (with the understanding that the French side provides the support for subsistence; it would be desirable to fix this in the formal agreement with the French side). Also, travel and subsistence for short visits within the UK will be supported. Upon request to the PI, and after consultation with the steering committee, full support (i.e., travel plus subsistence) will be provided for a limited number of visits of UK early career researchers to France.

● **Week-Long Conferences.**

The network will organise three week-long conferences:

– 2008 (or 2010, if the timing is too tight): An interdisciplinary conference bringing together many people from the different areas of representation theory involved in the network ($\approx$ 100 participants). A possible place to hold such a conference is the conference center on the Isle of Skye.

– 2009: An instructional conference related to the Isaac Newton Programme "Algebraic Lie Theory" ($\approx$ 50 participants, majority from UK). The current plan is to center this around "Geometrization and Categorification", so that it provides background for a satellite conference (within the framework of the Newton programme) with the same title.

– 2010: A joint French-UK instructional conference at CIRM ($\approx$ 50 participants in total); the current suggestion is to center this conference around the Langlands Programme.

● **Invited Lecture Series in the UK.**

Every year, the network supports up to two invited lecture series by French speakers visiting a UK mathematics department. It is expected that the UK departments in question contribute to travel and subsistence of the French speaker. The network provides support for travel and subsistence for UK participants.

● **Participation of UK Researchers in French GDR Meetings.**

Every year (and independently of this network), both GDR's in France which will be twinned with this network organise a week-long workshop at the CIRM conference center in Luminy (France). For example, the GDR 2249 of B. Rémy will organise a meeting on "Hecke algebras, Groups and Geometry", 13–17 October, 2008. The PI is a coorganiser of this meeting, the CoI and Shaun Stevens will give short lecture courses. Together with the other organisers, it was agreed to reserve a number of further places for UK participants, within the framework of this network. (It would be desirable to fix similar arrangements for the subsequent meetings in the formal agreement with the French side.) The available funds at CIRM may be limited; the network provides support for travel and subsistence for UK participants.

● **Annual Short Meetings in the UK.**

The network organises every year four short meetings (1-2 days), widely spread geographically and scientifically. At least two speakers in each meeting should be from France. The network provides support for travel and subsistence for the French speakers and UK participants.

● **Related Events.**

The network gives access to and provides support for travel and subsistence of UK researchers wishing to attend events related to representation theory, e.g., the "Summer School on Geometric Methods in Representation Theory" in Grenoble 2008 and the planned conference "Triangulated Categories II" in Newcastle 2009.

● **Public Interest Lectures.**

Each year, the network will hold at least one public interest lecture in the UK on representation theory. Support for travel will be provided for network partners. Speakers should be generally available either from within the UK community or from major events like the Isaac Newton Institute programme in 2009.

Relevance to Beneficiaries

The principal beneficiaries of this proposal are early career researchers in representation theory in the UK, where the term "representation theory" is understood in a broad sense. Instructional conferences will help to train young early-stage researchers by broadening the canvas on which they work, lead to new collaborations and generate future activity in key fields of research. New contacts, both in the UK and in France, will be facilitated through regular exchanges and the short-visit scheme. Opportunities are provided to meet experts in areas where France is particularly strong (e.g., the representation theory of p-adic groups and the Langlands programme). Of course, the same remarks apply to established representation theorists in the UK and, indeed, through the various connections of representation theory with other areas of mathematics, to the broader mathematical community. The new collaborations are expected to result in genuine advances in representation theory.

Dissemination and Exploitation

This will be mainly through regular conferences, public interest lectures, and a network webpage containing lists of members with interests, forthcoming meetings, links to earlier meetings with lots of active hyperlinks and notes, etc. The various visits and further exchanges can be expected to lead to significant research output.

Management of the Network

The PI will be responsible for the overall management of the programme working closely with the CoI, and assisted by an administrator to be appointed. The duties of the PI and CoI include:

- coordinating annual invited lecture series, public interest lectures and short meetings, as detailed above (with the understanding that the actual organisation is done by the UK departments concerned);

- coordinating the organisation of the annual conferences;

- coordinating the visits of early researchers between France and UK;

- maintaining a webpage containing lists of members with interests, information about forthcoming meetings, links to earlier meetings with lots of active hyperlinks and notes, etc.

Matters or a mere administrational nature (e.g., refunding of expenses for travel and subsistence) will be handled by the administrator, through the administration of Aberdeen University.

There will be a **scientific steering committee**, the composition of which should be balanced with respect to geographic representation of the network and the various branches of representation theory involved in it. The committee will meet at least once each year and review the activities of the network. Thus, the committee will

- encourage network partners to run programmes in targeted areas;

- advise members applying for visits from the UK and France;

- resolve conflicting situations (e.g., balance of visits with respect geographic representation and the various branches of representation theory);

- oversee the scientific content and balance of the network's activities.

Initially, the members of the steering committee are:

Meinolf Geck (PI)
Iain Gordon (CoI)
Joseph Chuang (Bristol)
John Greenlees (Sheffield)
Raphael Rouquier (Oxford)
Shaun Stevens (East Anglia)

At the end of the first year of its existence, the steering committee will review its own composition and enlarge or renew where appropriate. Once the network has been approved, and the cooperation with the French side been formalised, the French network partners will also be represented on the steering committee.

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