Triangulated Categories in Algebra, Geometry and Topology

Abstracts

Lecture series

Alexey Bondal (Steklov Mathematical Institute, Moscow)
Monday 11:00–11:50; Tuesday 17:00–17:50; Wednesday 11:30–12:20

Categorical constructions with application to algebraic geometry

We will discuss old and new constructions in the theory of abelian and triangulated categories, such as localizing subcategories, torsion pairs, glued t-structures, spherical functors, etc. in view of applications to algebraic geometry and representation theory. We will explain the topological analogy that gives a categorical insight into birational geometry and show how it works on some aspects of homological interpretation of the Minimal Model Program in birational geometry such as blow-ups and flops.

Joseph Chuang (City University, London)
Monday 15:30–16:20; Tuesday 15:30–16:20; Wednesday 17:00–17:50

Perverse equivalences

These lectures will be an introduction to perverse equivalences, certain special derived equivalences between abelian categories that Raphael Rouquier and I have studied. Some of the examples of perverse equivalences I will discuss are related to Broué's Abelian defect group conjecture in the representation theory of finite groups, categorical actions of groups and Lie algebras, and auto-equivalences such as spherical twists and P-twists.

Kathrin Hess Bellwald (École Polytechnique Fédérale de Lausanne)
Monday 12:00–12:50; Monday 17:00–17:50; Tuesday 11:30–12:20

Homotopical Morita theory for corings, with applications to Hopf-Galois theory

(Joint work with Alexander Berglund) A coring (A,C) consists of an algebra A in a symmetric monoidal category and a coalgebra C in the monoidal category of A-bimodules. Corings and their comodules arise naturally in the study of Hopf-Galois extensions and descent theory, as well as in the study of Hopf algebroids. In this series of lectures, I will address the question of when two corings (A,C) and (B,D) in a symmetric monoidal model category $\mathcal{V}$ are homotopically Morita equivalent, i.e., when their respective categories of comodules $\mathcal{V}_A^C$ and $\mathcal{V}_B^D$ are Quillen equivalent.

The category of comodules over the trivial coring (A,A) is isomorphic to the category $\mathcal{V}_A$ of A-modules, so the question englobes that of when two algebras are homotopically Morita equivalent. I will begin by discussing this special case, extending previously known results.

To approach the general question, I will introduce the notion of a *braided bimodule* and show that adjunctions between $\mathcal{V}_A$ and $\mathcal{V}_B$ that lift to adjunctions between $\mathcal{V}_A^C$ and $\mathcal{V}_B^D$ correspond precisely to braided bimodules. I will describe descent-type criteria for when a braided bimodule induces a Quillen equivalence between $\mathcal{V}_A^C$ and $\mathcal{V}_B^D$. In particular, I will provide conditions under which a morphism of corings induces a Quillen equivalence, providing a homotopic generalization of results by Hovey and Strickland on Morita equivalences of Hopf algebroids. As an illustration of the general theory, I will describe in detail the homotopical Morita theory of corings in the category of chain complexes over a commutative ring.

I will conclude by revisiting the theory of homotopic Hopf-Galois extension, in light of the homotopical Morita theory of comodules. In particular, I will discuss homotopic Hopf-Galois extensions of differential graded algebras over a commutative ring, for which we establish a descent-type characterization analogous to the one Rognes provided in the context of ring spectra. An interesting feature in the differential graded setting is the close relationship between homotopic Hopf-Galois theory and Koszul duality theory. Moreover, nice enough principal fibrations of simplicial sets give rise to homotopic Hopf-Galois extensions in the differential graded setting, for which this Koszul duality has a familiar form.

Bertrand Toën (Université de Toulouse)
Tuesday 10:30–11:20; Wednesday 10:30–11:20; Wednesday 15:30–16:20

Dg-categories and motives

The purpose of this series of lectures is to present the construction of a motive associated to a dg-category as well as some applications. The first lecture will be devoted to reminders about the homotopy theory of dg-categories, the notions of smoothness, properness and their interpretations using duality in symmetric monoidal 2-categories. In the second lecture I will present the construction of a motivic BU-module associated to a given dg-category, as well as some of its basic properties and examples. Finally, in the last lecture we will see how this general formalism can be used in order to prove an index formula in the l-adic setting. Some applications in the context of matrix factorisations and vanishing cycles will be given.

Plenary talks (in chronological order)

Osamu Iyama (Graduate School of Mathematics, Nagoya University)
Monday 09:30–10:20

Lattice structure of preprojective algebras and Weyl groups

Tilting theory of the preprojective algebra A of an acyclic quiver Q categorifies the corresponding Coxeter group W. When Q is Dynkin, there exists an isomorphism of lattices between W with the opposite weak order and torsion classes of A (that is, a full subcategory of $modA$ closed under factor modules and extensions). This gives bijections between join-irreducible elements in W and bricks of A (that is, A-modules X satisfying $End_A(X) = k$). If time permits, for type A, we characterize the lattice quotients of W coming from algebra quotients of A. This is a joint work with N. Reading, I. Reiten and H. Thomas.

Moritz Groth (Universität Bonn)
Tuesday 09:00–09:50

Characterizations of abstract stable homotopy theories

Stable derivators provide one of the many enhancements of triangulated categories, as is witnessed, for example, by the existence of canonical triangulations and related structures. We begin this talk by quickly reviewing these results, thereby setting the stage for the talk by Jan Šťovíček on applications in abstract representation theory.

The main focus of this talk, however, will be on various characterizations of stable derivators. Every such characterization specializes to an answer to the following question: what are the defining features of the passage from (pointed) topological spaces to spectra?

Alexander Kuznetsov (Steklov Mathematical Institute, Moscow)
Wednesday 09:00–09:50

Rectangular Lefschetz decompositions and fractional Calabi–Yau categories

In the talk I will discuss a general way to construct geometric examples of fractional Calabi–Yau categories, i.e. semiorthogonal components of derived categories of coherent sheaves on smooth projective varieties, such that a power of their Serre functor is isomorphic to a shift. One should start with a smooth projective variety and a so-called rectangular Lefschetz decomposition (with respect to some line bundle) of its derived category, and consider a divisor in it, corresponding to a power of this line bundle, or a double cover branched over such a divisor. Then its derived category has an induced semiorthogonal decomposition, with some components coming from the initial variety, and the addition component having this fractional Calabi–Yau property.

Ivo Dell'Ambrogio (Université de Lille 1)
Thursday 09:00–09:50

Affine weakly regular tensor triangulated categories

In general, the problem of classifying the thick subcategories of a triangulated category can be a very difficult one. This has been clear since the first results were obtained in the 80's and 90's by Devinatz, Hopkins and Smith in homotopy

theory, by Hopkins, Neeman and Thomason in algebraic geometry, and by Benson, Carlson and Rickard in modular representation theory. In the last decade, much progress has been made in formalizing the problem and the methods for solving it, especially when a tensor product is available. Notably, Balmer has developed a whole geometric theory of tensor triangulated categories and Benson, Iyengar and Krause have exploited the powerful context of compactly generated triangulated categories. In the examples, these theories often allow one to reduce the classification problem of thick (or localizing) subcategories to the study of some special local ones, which then can hopefully be understood by methods specific to the example at hand.

In this talk, I will present joint work with Don Stanley showing how these methods can sometimes go all the way to yield the desired classifications purely formally, without requiring extra knowledge of the specific examples. Sufficient hypotheses are that the tensor unit generates and its graded endomorphism ring is noetherian and satisfies some weak regularity properties.

Kiriko Kato (Osaka Prefecture University)
Thursday 10:30–11:20

Polygon of recollements

Polygons of recollements introduced in our previous work result in high symmetry of triangulated categories. In this talk, we study what cause polygons of recollements and give some examples related to N-complexes. Joint work with O. Iyama and J. Miyachi.

Raf Bocklandt (Universiteit van Amsterdam)
Thursday 11:30–12:20

Bands and Pants

For some categories of matrix factorizations one can describe the indecomposable objects in terms of strings and bands. Using mirror symmetry some of these bands can be interpreted as real bands on a surface. These bands can be used to cut the surface in pants and this can be related to stability conditions on the category of matrix factorizations.

Jan Šťovíček (Charles University in Prague)
Thursday 15:30–16:20

Abstract reflection functors

The talk will focus on a minimalistic enhancement of triangulated categories, so-called stable derivators. Several phenomena known from classical representation theory have a counterpart in this setting. Here we explain an abstract version of the Bernstein-Gelfand-Ponomarev reflection functors. Although this is in fact an analysis of certain very general aspects of abstract stable homotopy theory, it has a strong combinatorial and representation theoretic flavour. The reflection functors are in particular always represented by tilting bimodules, which in this case are certain representations in the category of spectra. This is an account on a joint work with Moritz Groth.

Ryo Takahashi (Graduate School of Mathematics, Nagoya University)
Thursday 17:00–17:50

Dimensions of derived categories of commutative rings

The notion of the dimension of a triangulated category has been introduced implicitly by Bondal and Van den Bergh, and explicitly by Rouquier. Roughly speaking, it measures how many exact triangles are necessary to build the triangulated category from a single object. In this talk, I will speak mainly about finiteness of dimensions of derived categories of commutative Noetherian rings.

Wendy Lowen (Universiteit Antwerpen)
Friday 09:00–09:50

The Gerstenhaber-Schack complex for prestacks and singular schemes

The starting point for this talk is the complex Gerstenhaber and Schack defined in the 1980ies for presheaves of algebras, in order to compute a natural notion of Hochschild cohomology. We explain how this complex describes de-

formations as a twisted presheaf, and how this relates to the HKR decomposition for smooth schemes. We also present an application of the complex to (possibly singular) projective hypersurfaces. We introduce a Gerstenhaber-Schack type complex for arbitrary prestacks, involving a modification of the differential, and we discuss the relation with the Hochschild complex of the Grothendieck construction. This allows us to endow our complex with an L_∞ structure. The talk is based upon joint work with Hoang Dinh Van, with Liyu Liu, and with both.

Wei Hu (Beijing Normal University)
Friday 10:30–11:20

Derived equivalences and homological dimensions

Homological dimensions, such as global dimensions and dominant dimensions, are important homological invariants of algebras, carrying certain crucial information of algebras. The notion of derived equivalence is a fundamental equivalence between algebras. Unfortunately, homological dimensions are not preserved under derived equivalences in general. In this talk, we shall discuss the following two questions:

(1) *Which nice class of derived equivalences (between arbitrary finite dimensional algebras) preserves homological dimensions?*
(2) *For which nice class of algebras, every derived equivalence always preserves certain homological dimensions?*

In particular, we shall show that all derived equivalences between cellular algebras preserve global dimensions, and also dominant dimensions provided the dominant dimensions are positive. This can be applied to calculate the global dimension and dominant dimension of q-Schur algebra blocks. This is based on joint works with C.C. Xi, M. Fang and S. Koenig.

Agnieszka Bodzenta (University of Edinburgh)
Friday 11:30–12:20

Spherical pair for a flop

Consider varieties X and X^+ related by flops $f\colon X \to Y$, $f^+\colon X^+ \to Y$ with fibers of dimension less than or equal to one. The null category for f is the category of sheaves on X with vanishing derived direct image. I will show that the derived categories of null categories for f and f^+ form a spherical pair in an appropriate quotient of the derived category of the fiber product of $X \times_Y X^+$. The associated auto-equivalence of the derived category of X is the flop-flop functor. This is a joint work with A. Bondal.

Steffen Oppermann (Norges Teknisk-Naturvitenskapelige Universitet - NTNU, Trondheim)
Friday 14:30–15:20

Quivers for silting mutation

Aihara and Iyama introduced silting mutation as a generalization of tilting mutation. The idea is similar as for tilting mutation: Replace one indecomposable summand of a silting object by a different indecomposable, which is obtained from an approximation. In this talk we ask how the (derived) endomorphism rings of silting objects change under mutation. More precisely I will give a combinatorial rule in terms of quivers. I will illustrate this rule on some simple (and well-known) examples.

Tobias Dyckerhoff (Hausdorff Center for Mathematics, Bonn)
Friday 16:00–16:50

Relative Calabi-Yau structures

The basic operation of oriented cobordism is to glue two oriented manifolds along a common boundary component to produce a new oriented manifold. In this talk, we discuss a generalization of this procedure to noncommutative geometry: we introduce the concept of a relative Calabi-Yau structure, defined on a functor of differential graded categories, which should be interpreted as an analog of an oriented manifold with boundary. As an application of the resulting theory, we construct relative Calabi-Yau structures on topological Fukaya categories of punctured framed Riemann surfaces. Time permitting, we discuss relations to derived symplectic geometry on moduli spaces of objects.

Based on joint work in progress with Chris Brav.