CLUSTER-HEARTS AND CLUSTER-TILTING OBJECTS

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Summary

Inspired by Kontsevich's homological mirror conjecture, Seidel and Thomas studied in [10] actions of the braid groups generated by the so-called *twist functors along spherical objects* on triangulated categories.

On the other hand, *cluster algebras* were invented at around 2000 by Fomin-Zelevinsky [3]. Their main motivation was to find a combinatorial approach to Lusztig's results concerning total positivity in algebraic groups [8] and canonical bases in quantum groups [7]. However, shortly after their appearance, strong links with other areas of Mathematics were discovered: Poisson geometry, discrete dinamical systems, algebraic geometry, representation theory of finite-dimensional algebras,...

Nowadays, a major effort is being made to understand cluster algebras by 'categorifying' them, namely, by finding nice categories encoding their combinatorics. There are three main types of categorification:

- additive, by means of the 2-Calabi-Yau triangulated *cluster category* (after Buan-Marsh-Reineke-Reiten-Todorov [2], Keller [5], Amiot [1],...),
- monoidal (after Hernandez-Leclerc [4], Nakajima [9],...), and
- a categorification by means of a 3-Calabi-Yau triangulated category, recently developped by Kontsevich-Soibelman [6], which is somewhere in the middle of the additive and the monoidal types.

In a joint work with Bernhard Keller, we use braid actions on triangulated categories, t-structures and weight structures to understand the relationship between Kontsevich-Soibelman categorification and the additive one.

References

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