Two weeks of silting – Conference
August 5 – August 9, 2019, University of Stuttgart
List of Abstracts

Silting-connected triangulated categories

Takuma Aihara

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We explore when a triangulated category is silting-connected. There are three levels of silting-connectedness: (i) silting-connected (ii) strongly silting-connected (iii) silting-discrete. We then have the implications (iii) $\Rightarrow$ (ii) $\Rightarrow$ (i) [A, AI]. In this talk, we give a list of (finite dimensional) algebras with silting-connected/discrete perfect derived category. On the other hand, we ask if strong silting-connectedness implies silting-discreteness; it still does not seem that a counterexample has been found. Moreover, we also discuss how to show that a given triangulated category is silting-discrete [AAC, AM]. Finally, we give an example of algebras whose perfect derived categories are not silting-connected [AGI].

References

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[AGI] Takuma Aihara, Joseph Grant and Osamu Iyama, Private communication.

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Reduction of the wall-chamber structures

Sota Asai

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This talk is based on [Asa]. Let A be a finite-dimensional algebra over a field K. Then, each element θ of the real Grothendieck group $K_0(\mathsf{proj}\,A)_\mathbb{R} := K_0(\mathsf{proj}\,A)\otimes_\mathbb{Z}\mathbb{R}$ of the category of finitely generated A-modules gives a stability condition in the sense of King [Kin]. As in [BST], for each nonzero module M in the category $\mathsf{mod}\,A$ of finite-dimensional A-modules, we can consider the subset $\Theta_M \subset K_0(\mathsf{proj}\,A)_\mathbb{R}$ consisting of all θ such that M is θ-semistable. We call the subset Θ_M the *wall* associated to the module M, and these walls define a wall-chamber structure of $K_0(\mathsf{proj}\,A)_\mathbb{R}$.

In this talk, I will mainly deal with reduction of the wall-chamber structures. For a 2-term presilting object U in $\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\,A)$ with its Bongartz completion T, we can consider the wide subcategory $\mathcal{W}_U := {}^{\perp}H^{-1}(\nu U) \cap H^0(U)^{\perp} \subset \mathsf{mod}\,A$ and the algebra $B := \mathsf{End}_{\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\,A)}(T)/[U]$. Then, *$\tau$-tilting reduction* [Jas] gives a category equivalence $\mathcal{W}_U \to \mathsf{mod}\,B$ and a one-to-one correspondence between the 2-term presilting objects in $\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\,A)$ containing U as a direct summand and the 2-term presilting objects in $\mathsf{K}^{\mathrm{b}}(\mathsf{proj}\,B)$. Motivated by this, I compared the two wall-chamber structures of $K_0(\mathsf{proj}\,A)_{\mathbb{R}}$ and $K_0(\mathsf{proj}\,B)_{\mathbb{R}}$, and obtained that the wall-chamber structure of $K_0(\mathrm{proj}\,B)_{\mathbb{R}}$ can be recovered from the local wall-chamber structure of $K_0(\mathsf{proj}\,A)_{\mathbb{R}}$ around the element $[U] \in K_0(\mathsf{proj}\,A)_{\mathbb{R}}$ via the restriction of a certain linear projection $\pi\colon K_0(\mathsf{proj}\,A)_{\mathbb{R}} \to K_0(\mathsf{proj}\,B)_{\mathbb{R}}$. I will talk about its detail.

References

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Stability conditions, contraction algebras and $K(\pi, 1)$

Jenny August

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For a finite dimensional algebra, Bridgeland stability conditions can be viewed as a continuous generalisation of silting complexes. If the algebra is silting-discrete then Pauksztello–Saorín–Zvonareva show that the resulting space of stability conditions is contractible. In this talk, I'll look at a special class of silting-discrete symmetric algebras, known as contraction algebras, and show how the nice behaviour of their silting theory allows us to describe their stability manifold as the universal cover related to a hyperplane arrangement. I'll also explain how this result in homological algebra provides a new proof of a classically topological result, known as the $K(\pi, 1)$ conjecture.

Cotilting with balanced big Cohen–Macaulay modules

Isaac Bird

University of Manchester

Over a Cohen-Macaulay local ring the class of balanced big Cohen–Macaulay modules, as introduced by Hochster, provides an intuitive way to extend the class of maximal Cohen–Macaulay modules beyond finitely generated modules. We consider the definable closure of this class, show that it is cotilting and use this to describe the minimal injective resolutions of balanced big Cohen–Macaulay modules. This furthers a classic result of R. Sharp. We then illustrate how, in certain cases, this result extends beyond the local case and give an application for Gorenstein flat modules over commutative Gorenstein rings. We contrast this situation by considering the cotilting class formed by taking modules of maximal Ext depth. We give a description of this class and its cotilting structure, including providing a cotilting module.

The ascent-descent property for 2-term silting complexes

Simion Breaz

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Let $\lambda : R \to S$ be a homomorphism of unital rings. A property $\mathcal{P}$ associated to a complex of modules *ascends* along λ if the functor $- \otimes_R S$ preserves the property $\mathcal{P}$, i.e. for every complex $\mathbf{C} : \cdots \to C^i \to C^{i+1} \to \cdots$ of right R-modules which satisfies $\mathcal{P}$, the complex of S-modules $\mathbf{C} \otimes_R S : \cdots \to C^i \otimes_R S \to C^{i+1} \otimes_R S \to \cdots$ has the property $\mathcal{P}$. Conversely, $\mathcal{P}$ *descends* along λ if a complex $\mathbf{C}$ in Mod-R satisfies $\mathcal{P}$ provided that $\mathbf{C} \otimes_R S$ satisfies $\mathcal{P}$ as a complex of right S-modules. These notions are natural extensions of the corresponding ascent/descent notions associated to module properties. The properties of modules which ascend along flat ring homomorphisms and descend along faithful flat ring homomorphisms (called *ascent-descent* properties) play an important role in commutative algebra since the corresponding properties associated to quasi-coherent sheaves have a local character. For instance, for modules over commutative rings the properties "projective" (Gruson, M. Raynaud) and "1-tilting" (Hrbek, Šťovíček, Trlifaj) are ascent-descent.

In this talk I will present results about the ascent/descent properties for 2-*term silting complexes*. These include, for the general case, a characterization for the ascending property of 2-term silting complexes. For the commutative case we will see that the ascending property of 2-term silting complexes is valid along all ring homomorphisms. Moreover, the silting property associated to 2-term complexes descends along faithfully flat ring homomorphisms of commutative rings. In the end I will discuss about some possible continuations of this research for silting complexes of arbitrary length or for cosilting complexes.

τ-exceptional sequences and wide subcategories

Aslak Bakke Buan

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This is based on joint work with Robert Marsh [1, 2].

We define (signed) τ-exceptional sequences over arbitrary finite dimensional algebras, as a natural generalization of exceptional sequences over hereditary algebras. In particular, generalizing work of Igusa and Todorov [3], we show that there is a correspondence between signed τ-exceptional sequences and ordered support τ-tilting modules. Restricting to the case of τ-tilting finite algebras, we also show how this concept turns up naturally when defining a categorical structure on the set of wide subcategories of a given module category,

References

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[3] K. Igusa and G. Todorov, *Signed exceptional sequences and the cluster morphism category*, preprint, arXiv:1706.02041 [math.RT]

Simple-minded reduction

Raquel Coelho Simões

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Stable module categories of selfinjective algebras are an important class of triangulated categories. The central open problem for stable module categories is the Auslander–Reiten conjecture, which states that stable equivalences preserve the number of isomorphism classes of non-projective simple modules. The study of this conjecture led to the notion of simple-minded systems. This notion generalises the concept of simple modules, which are the natural generators of stable module categories, given the absence of projective modules.

In this talk, we discuss the notion and properties of simple-minded systems in the setting of negative Calabi–Yau triangulated categories. A motivating class of examples of such categories are the stable module categories of symmetric algebras, which are -1 Calabi–Yau. We will also describe a reduction technique for constructing these objects inductively, and time permitting, we will consider analogues of our results for simple-minded collections.

This will be a report on joint work with David Pauksztello and on joint work in progress with David Pauksztello and Alexandra Zvonareva.

More finite sets coming from non-commutative counting

George Dimitrov

University of Vienna

In a joint work with Katzarkov we introduced categorical invariants which are, roughly speaking, sets of triangulated subcategories in a given triangulated category and their quotients.

In this talk I will sketch the definition of non-commutative counting invariants, structures on them, functoriality, and examples. Intersection of the entities I count will be illustrated, time permitting.

Examples of algebras that are not silting connected

Alex Dugas

University of the Pacific

A finite-dimensional algebra A is said to be silting connected if every basic silting object in $K^b(\text{proj-}A)$ can be obtained from A by iterated irreducible silting mutation. While this is known to be the case for several classes of algebras, including representation finite symmetric algebras and piecewise hereditary algebras, an example of Aihara, Grant and Iyama reveals that not all algebras are silting connected. However, this example still satisfies a weaker form of silting connectivity, as all silting objects in $K^b(\text{proj-}A)$ can be obtained from A by iterated silting mutation if one includes reducible mutations. In this talk we give examples of algebras where even this weaker form of silting connectivity fails. Our argument is based on the fact that silting mutation preserves invariance under twisting by a fixed algebra automorphism, combined with the existence of spherical modules that are not invariant under such a twist.

Silting complexes over Noetherian algebras modulo a central regular sequence

Wassilij Gnedin

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Let Λ be a ring such that its center R is Noetherian local and complete, and the module ${}_{\mathsf{R}}\Lambda$ is finitely generated. Assume that Λ has a regular sequence (for example, one non-zerodivisor) $\mathbf{x}$ contained in the maximal ideal $\mathfrak{m}$ of R.

My talk is guided by the following question:

> *How is the derived representation theory of the Noetherian $\mathbf{x}$-regular R-algebra Λ different from that of its quotient ring $\mathsf{A} = \Lambda/\mathbf{x}\Lambda$?*

A commutative version of this question was studied by Avramov and Buchweitz, Eisenbud and Yoshino. It is related to the problem to lift a given complex of projective A-modules to some complex of projective Λ-modules.

In my talk, I will present three results on the question above:

(a) *There is a bijection of isomorphism classes* $\mathsf{presilt}\,\Lambda \overset{1:1}{\longleftrightarrow} \mathsf{presilt}\,\mathsf{A}$ *which preserves the partial order and the silting property.*

(b) *There is a bijection* $\{\, T_\bullet \in \mathsf{tilt}\,\Lambda \mid \mathrm{End}(T_\bullet)$ *is* $\mathbf{x}$*-regular* $\} \overset{1:1}{\longleftrightarrow} \mathsf{tilt}\,\mathsf{A}$.

(c) *For any ring* B *which is derived equivalent to the quotient ring* A *there is a Noetherian* $\mathbf{x}$*-regular* R*-algebra* Γ *such that* $\mathsf{B} \cong \Gamma/\mathbf{x}\Gamma$.

A result similar to (a) was shown independently by Kimura and Mizuno [1].

My approach to prove the preceding results uses Rickard's lifting techniques.
In the case that the module ${}_{\mathsf{R}}\Lambda$ is free of finite rank and A is replaced by $\Lambda/\mathfrak{m}\Lambda$, analogues of (b) and (c) have been proven by Rickard in [2] and [3].

At the end of my talk, I will discuss some applications of the results above.

References

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Preprojective algebras and derived Picard groups

Joseph Grant

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I will describe some work in progress on a relation between derived Picard groups of (higher) hereditary algebras and automorphism groups of (higher) preprojective algebras. This gives a useful criterion to check if an algebra is fractionally Calabi–Yau.

Cluster tilting modules for mesh algebras

Sira Gratz

University of Glasgow

Cluster tilting modules for finite dimensional algebras are notoriously elusive, and their existence has powerful consequences for the representation theoretic properties of such an algebra.

We present a new example of naturally occurring self-injective algebras that have cluster tilting modules: The mesh algebras of non-simply laced Dynkin type. In this talk, we will discuss how we can exploit work by Darpö and Iyama, and by Geiss, Leclerc and Schröer, to show the existence of cluster tilting modules. Moreover, we will discuss the importance of a specific automorphism of mesh algebras, and show how we can use a twisted Ext-symmetry to describe mutation of cluster tilting modules. This provides an explicit example of mutation of cluster tilting modules in a module category that is not stably 2-Calabi–Yau.

This talk is based on joint work with Karin Erdmann and Lisa Lamberti.

Homotopically smashing t-structures over commutative noetherian rings

Michal Hrbek

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This is a report on a recent joint work with Tsutomu Nakamura. We show that, in the unbounded derived category of a commutative noetherian ring, any homotopically smashing t-structure is generated by a set of compact objects. As a consequence, such t-structures are subject to the classification due to Alonso Tarrío, Jeremías López and Saorín.

The notion of homotopically smashing t-structure in a (strong, stable) Grothendieck derivator was introduced in a recent paper by Saorín, Šťovíček, and Virili as a t-structure such that its coaisle is closed under all directed homotopy colimits. For a general t-structure, this condition is strictly stronger then requiring just the closure under coproducts. For stable t-structures however, the homotopically smashing condition is equivalent to the usual smashing property of the induced Bousfield localization functor. In this way, our result generalizes the telescope conjecture due to Neeman. In a different direction, restricting to non-degenerate t-structures, our result recovers the cofinite type of n-cotilting modules due to Angeleri, Pospíšil, Šťovíček, and Trlifaj, and generalizes it to arbitrary pure-injective cosilting objects in the derived category.

The green groupoid and its action on derived categories (joint work with Milen Yakimov)

Peter Jørgensen

Newcastle University

We introduce the *green groupoid* $\mathcal{G}$ of a 2-Calabi–Yau triangulated category $\mathcal{C}$. It is an augmentation of the mutation graph of $\mathcal{C}$, which is defined by means of silting theory.

The green groupoid $\mathcal{G}$ has certain key properties:

1. If $\mathcal{C}$ is the stable category of a Frobenius category $\mathcal{E}$, then $\mathcal{G}$ acts on the derived categories of the endomorphism rings $\mathcal{E}(m, m)$ where m is a maximal rigid object.
2. $\mathcal{G}$ can be obtained geometrically from the g-vector fan of $\mathcal{C}$.
3. If the g-vector fan of $\mathcal{C}$ is a hyperplane arrangement $\mathcal{H}$, then $\mathcal{G}$ specialises to the Deligne groupoid of $\mathcal{H}$, and $\mathcal{G}$ acts faithfully on the derived categories of the endomorphism rings $\mathcal{E}(m, m)$.

The situation in (3) occurs if $\Sigma_{\mathcal{C}}^2$, the square of the suspension functor, is the identity. It recovers results by Donovan, Hirano, and Wemyss where $\mathcal{E}$ is the category of maximal Cohen–Macaulay modules over a suitable singularity.

Preprojective algebras of non-Dynkin type and two-term tilting complexes

Yuta Kimura

University of Bielefeld

The tilting theory of a preprojective algebra of Dynkin type has been studied well. For example, Aihara and Mizuno [AM] classified all tilting complexes of the algebra by using braid groups. If a preprojective algebra is of non-Dynkin type, in [BIRS, IR], an important family of tilting ideals over the preprojective algebra has been studied. Such tilting ideals are parameterized by the corresponding Coxeter group.

In this talk, we study two-term tilting complexes of a preprojective algebra of non-Dynkin type. By using the tilting ideals, we first observe that the Coxeter group parameterizes two families of two-term tilting complexes. Moreover, in the case of affine type, we show that any two-term silting complex belongs to one of them.

This is a joint work with Yuya Mizuno.

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A generalization of the Nakayama functor

Sondre Kvamme

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Let Λ be a finite-dimensional algebra over a field k. The algebra Λ is called Iwanaga–Gorenstein if the Λ-module $D(\Lambda) := \mathrm{Hom}_k(\Lambda, k)$ is tilting. In this case the projective dimension of $D(\Lambda)$ as a left and a right Λ-module is equal. This notion can also be described just using the Nakayama functor $\nu = D(\Lambda) \otimes_\Lambda -$.

In this talk we will introduce a generalization of the Nakayama functor by considering its interaction with the forgetful functor to vector spaces. We will show that it can be characterized in terms of an ambidextrous adjunction of monads and comonads. Furthermore, we will introduce a generalization of finite-dimensional Iwanaga–Gorenstein algebras, and in particular, we will obtain an analogous result to the equality of the projective dimensions of $D(\Lambda)$. The talk will be illustrated on specific examples.

Derived equivalences of Jacobian algebras

Sefi Ladkani

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It is known that the Ginzburg dg-algebras of two quivers with potentials related by a mutation are derived equivalent. While an analogous statement for the corresponding Jacobian algebras is not always true, such situation nevertheless gives rise to four silting complexes, two for each algebra.

In the first part of the talk I will formulate the conditions under which such a complex becomes tilting, and explain how by coupling a pair of these tilting complexes (one for each algebra) one gets the derived equivalence. I will present two applications; the first is a decision algorithm for derived equivalence given the Cartan matrices of the algebras, and the second is about some consequences in the weakly symmetric case.

In the second part I will demonstrate how global reasoning based on the underlying combinatorics can settle the problem of derived equivalence classification of Jacobian algebras for various mutation classes of quivers with potentials. Some of the combinatorics involved concerns triangulations of surfaces and the corresponding ribbon graphs.

Cosilting modules over cluster-tilted algebras of type $\tilde{A}$

Rosanna Laking

University of Verona

Cosilting modules over a finite-dimensional algebra Λ simultaneously generalise the notion of support τ^{-1}-tilting modules and (large 1-) cotilting modules. In fact, the finite-dimensional cosilting modules are exactly the support τ^{-1}-tilting modules and the faithful cosilting modules are exactly the cotilting modules. In general, the cosilting modules are infinite-dimensional, nevertheless, they are an important tool for understanding the structure of the category of finite-dimensional modules since they parametrise *all* torsion pairs in $\operatorname{mod}\Lambda$.

In this talk I will report on joint work with Karin Baur, in which we give a characterisation of cosilting modules in terms of certain 'maximal rigid' sets of indecomposable pure-injective modules. Using this characterisation we classify the cosilting modules over cluster-tilted algebras of type $\tilde{A}$ and show that they are in bijection with asymptotic triangulations of the annulus.

Higher τ-tilting and stratifications of higher cluster-tilting subcategories

Jordan McMahon

University of Graz

Higher τ-tilting is much more difficult to study than in the classical case, since the notions of support and filtrations are not generally understood for higher cluster-tilting subcategories. The class of higher representation-finite algebras should be understood as analogues of hereditary algebras; just as quasi-hereditary algebras (and standardly-stratified algebras) are. We introduce stratifications of higher cluster-tilting subcategories, and use examples to show they are naturally abundant for d-representation-finite algebras. We show how higher torsion classes can be defined with respect to a stratification, and explain how higher τ-tilting modules, higher torsion classes and wide subcategories can be related in this setting.

Co-t-structures cogenerated by weak cocompact objects

George Ciprian Modoi

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Let $\mathcal{T}$ be a triangulated category. If $\mathcal{T}$ has products and we start with a set $\mathcal{C}$ of compact objects in $\mathcal{T}$, then the construction of a t-structure whose aisle is the smallest containing $\mathcal{C}$ tends to be well-known (see [1]). Extending some results in [2], we perform a dual construction. Suppose $\mathcal{T}$ has products. Since non-zero cocompact objects usually do not exist, in the cited paper is defined a weaker notion, called 0-cocompactness. We modify this notion and we call weak cocompact the objects defined in this way. Starting with a set $\mathcal{C}$ of weak cocompact objects we construct a co-t-structure cogenerated by $\mathcal{C}$. A consequence regarding Brown representability for the dual is also considered.

References

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Faithfulness of ADE braid group actions on triangulated categories

Anna Nordskova

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Let Γ be a simply-laced Dynkin diagram and $\mathfrak{D}$ an enhanced triangulated category. A *spherical twist* [1] is an autoequivalence of $\mathfrak{D}$ constructed from an ω-spherical object, i.e. an object of $\mathfrak{D}$ whose self-Ext algebra is the same as the cohomology of the ω-sphere. Seidel and Thomas showed that spherical twists along a so-called Γ-configuration of ω-spherical objects satisfy braid relations of type Γ modulo natural isomorphisms, hence induce an action of an Artin group (generalised braid group) B_Γ on $\mathfrak{D}$.

Various results on the faithfulness of this action have been obtained over the years (e.g. for $\Gamma = A_n$ or $w = 2$, and in some other cases). In this talk, we present a new approach that allows us to establish faithfulness for any ADE diagram, any enhanced triangulated category and any $\omega \neq 1$ simultaneously. A crucial role in our method is played by two-term objects, the generalisation of two-term partial tilting complexes to the setting of triangulated categories with a Γ-configuration of spherical objects.

This is joint work with Yury Volkov.

References

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A derived equivalence classification of Brauer graph algebras

Sebastian Opper

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I will report on a joint project with Alexandra Zvonareva. Recently, gentle algebras have been classified up to derived equivalence in terms of their associated surface models. Based on these results, I will explain how covering theory of Brauer graph algebras and the relationship of gentle algebras and Brauer graph algebras can be exploited to obtain a complete derived invariant for the class of Brauer graph algebras.

Silting pairs and the stability manifold of discrete derived algebras

David Ploog

Leibniz University of Hannover

The stability manifold of a triangulated category is an interesting invariant but generally very hard to compute. In this talk, we show how the poset of silting pairs can be used to proved that the stability manifold of a discrete derived algebra is contractible.

Cosilting objects

Flaviu Pop

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In this talk, there are presented the cosilting objects such as cosilting modules and cosilting complexes. Cosilting modules were introduced as both a generalization of cotilting modules and as a dualization of silting modules, while cosilting complexes are defined as duals of silting complexes. Throughout this talk, there are presented basic properties of these notions and the relationship between them.

The tilting-cotilting correspondence

Leonid Positselski

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The infinitely generated n-tilting theory is formulated in the categorical context as a bijective correspondence between complete, cocomplete abelian categories A with an injective cogenerator J and an n-tilting object T and complete, cocomplete abelian categories B with a projective generator P and an n-cotilting object W. Then there is a derived equivalence $\mathsf{D}^\star(\mathsf{A}) \cong \mathsf{D}^\star(\mathsf{B})$. The Wakamatsu ($n = \infty$) case can be also included, leading to so-called *pseudo-derived equivalences* between the abelian categories A and B. The abelian category B can be described as the category of algebras/modules over the monad $X \longmapsto \mathrm{Hom}_{\mathsf{A}}(T, T^{(X)})$ on the category of sets. When A is a locally finitely generated Grothendieck abelian category (e.g., the category of modules over a ring), the abelian category B is the category of contramodules over the topological ring of endomorphisms of the tilting object T in A.

This talk is based on a joint work of Jan Šťovíček and the speaker.

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Explicit right adjoints between homotopy categories via "cocompact" objects

Chrysostomos Psaroudakis

University of Thessaloniki

Let T be a triangulated category with coproducts and let X be a set of compact objects. Then X generates a certain t-structure, and in particular describes explicitly a left adjoint to the inclusion of the coaisle. Unfortunately, it does not make much sense to consider the naive dual of this setup; cocompact objects rarely appear in categories which occur naturally. Motivated by this, we introduce a weaker version of cocompactness called 0-cocompactness, and show that in a triangulated category with products these objects cogenerate a t-structure. As an application, we provide explicit right adjoints between certain homotopy categories (i.e. "big" singularity categories). This is joint work with Steffen Oppermann and Torkil Stai.

Locally finitely presented and locally coherent hearts of t-structures

Manuel Saorín

University of Murcia

We will consider the Happel–Reiten–Smalø (HRS) t-structure in the (unbounded) derived category $\mathcal{D}(\mathcal{G})$ associated to a torsion pair $\mathbf{t} = (\mathcal{T}, \mathcal{F})$ in a Grothendieck category $\mathcal{G}$. By results of the speaker and Carlos Parra, it is known that the heart $\mathcal{H}_{\mathbf{t}}$ of that t-structure is a Grothendieck category if, and only if, $\mathbf{t}$ is of finite type, i.e., if and only if $\mathcal{F}$ is closed under taking direct limits in $\mathcal{G}$. In this talk we will try to identify the torsion pairs of finite type in $\mathcal{G}$ for which $\mathcal{H}_{\mathbf{t}}$ is locally finitely presented or locally coherent. We will show connections of this problem with the existence of certain cosilting objects, and will try to understand the relationship between cosilting and quasi-cotilting objects in an arbitrary Grothendieck category.

Smoothness of derived categories of algebras

Olaf Schnürer

University of Paderborn

We prove smoothness in the dg sense of the bounded derived category of finitely generated modules over any finite-dimensional algebra over a perfect field, hereby answering a question of Iyama. More generally, we prove this statement for any algebra over a perfect field that is finite over its center and whose center is finitely generated as an algebra. These results are deduced from a general sufficient condition for smoothness. This is joint work with Alexey Elagin and Valery Lunts.

Resolution and realisation functors

Nico Stein

University of Stuttgart

For a tilting complex T over a ring R, one can use a (bounded) realisation functor to obtain Rickard's equivalence between the bounded derived categories $\mathrm{D}^{\mathrm{b}}(\text{Mod-End}(T))$ and $\mathrm{D}^{\mathrm{b}}(\text{Mod-}R)$. We show that for a silting complex S over a ring R, there exists an (unbounded) realisation functor $\mathrm{D}(\mathcal{H}) \to \mathrm{D}(\text{Mod-}R)$ which has a left-adjoint, where $\mathcal{H}$ is the heart of the t-structure associated to S. If S is compact, then $\mathcal{H}$ is equivalent to Mod-End(T). Moreover, the left-adjoint can be explicitly described using a resolution functor (=strong weight complex functor) of the w-structure (=weight structure=co-t-structure) adjacent to the t-structure. Restricting this adjunction to the abelian level yields a silting theorem for 2-term complexes which correspond to silting modules.

A dual version for cosilting complexes is also true since the constructions can be carried out in the general setting of w-(co)silting objects in suitable derived categories. This treatment also allows us to deduce a Morita theorem for derived categories of suitable abelian categories which include left-complete Grothendieck categories in terms of w-cotilting objects.

From τ-tilting theory to stratifying systems

Hipolito Treffinger

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In this talk we will start by giving a brief overview on the path leading from quasi-hereditary algebras to stratifying systems. Then, after the main definitions are introduced, we will give a constructive proof that every τ-rigid module induces at least one stratifying system. Time permitting, we will show that every stratifying system found in this way is a τ-exceptional sequence, as recently introduced by Buan and Marsh. This is a joint work with Octavio Mendoza (arXiv:1904.11903).

Testing for projectivity, and transfinite extensions of simple artinian rings

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Baer Criterion of injectivity is a simple, but useful tool for classification of injective modules. Attempts to dualize it in order to test for projectivity include the Dual Baer Criterion [1], Whitehead test modules for projectivity [3], and more in general, p-test epimorphisms. All of these are available for perfect rings, but it is consistent with ZFC + GCH that none of these is available for any non-perfect ring [2], [3]. However, if R is not perfect and all flat R-modules have finite projective dimension (e.g., when R is a commutative noetherian ring of finite Krull dimension), then the existence of Whitehead test modules for projectivity, and hence of p-test epimorphisms, is indeed consistent with (and hence independent of) ZFC + GCH. As for the Dual Baer Criterion, it is known to fail for many classes of non-perfect rings (e.g., the commutative noetherian ones). However, as proved recently, the Dual Baer Criterion is consistent with (and hence independent of) ZFC + GCH for all small transfinite extensions of simple artinian rings [4], [5].

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Silting dg epimorphisms

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Bireflective subcategories of abelian and triangulated categories have long been studied in representation theory. In module categories, for example, they are well-known to be parametrised by equivalence classes of ring epimorphisms, and to be precisely those subcategories which are closed under kernels, cokernels, products and coproducts ([2, 3]). Similarly, in the derived category of a small dg category over a field, bireflective subcategories are parametrised by equivalence classes of homological dg epimorphism ([6]). In this talk, we describe bireflective subcategories of such derived categories in terms of closure conditions. Moreover, we observe that partial silting objects give rise to homological dg epimorphisms and that, often, every such epimorphism of *finite type* (i.e., the result of a localisation at a set of compacts) arises in this way. This is a derived version of the fact that universal localisations are silting ring epimorphisms ([5]). This is based on joint work with R. Laking ([4]), and on joint work with L. Angeleri Hügel and F. Marks ([1]).

References

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Silting, Happel–Reiten–Smalø tilting, and derived equivalences

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This talk is based on joint work [CHZ] with Xiao-Wu Chen and Zhe Han. We give necessary and sufficient conditions for a realization functor of a Happel–Reiten–Smalø (HRS) tilting to be an equivalence. The key idea is from the surjective algebra homomorphism from an algebra to the endomorphism algebra of an induced 2-term silting complex constructed in my joint work [BZ] with Aslak Bakke Buan. As an application, we show that if the HRS tilting is induced from a 2-term silting complex S, then the realization functor is an equivalence if and only if S is tilting.

References

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