Workshop on Orthogonal modules and homological dimensions

Titles and abstracts

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Series of four talks on orthogonal modules

Abstract:
Tachikawa's second conjecture predicts that a finitely generated, orthogonal module over a finite-dimensional self-injective algebra is projective. This conjecture is an important part of the Nakayama conjecture. In this series of lectures, we try to understand Tachikawa's second conjecture by addressing finitely generated, orthogonal generators over a self-injective algebra from the view point of triangulated categories.

Orthogonal modules I.
We recall the Nakayama conjecture and Tachikawa's two conjectures as well as some advances of these conjectures, and then state our main results on orthogonal modules. In particular, equivalent characterizations of Tachikawa's second conjecture in terms of relative Gorenstein categories or stratifying ideals of algebras.

Orthogonal modules II.
We establish a recollement of the relative stable categories for an orthogonal generator, and then describe compact objects of the right term of the recollement. This is based on a general construction of two pairs of triangle endofunctors for stable module categories. Some ideas of the proofs of the results mentioned in Part I will be presented.

Orthogonal modules III.
We introduce Gorenstein-Morita algebras and show that the Nakayama conjecture holds true for them, and then introduce (reduced) mirror-reflective algebras and establish a series of recollements of their derived module categories.

Orthogonal modules IV.
We describe explicitly the mirror-reflective algebras for algebras presented by quivers with relations, and then develop some homological properties of mirror-reflective algebras of gendo-symmetric algebras over fields.

Haruhisa Enomoto (Osaka Metropolitan University)

Orthogonal modules and (projectively) Wakamatsu tilting modules

Abstract:
Wakamatsu tilting modules are a special type of orthogonal module that generalize both tilting and cotilting modules, and naturally arise in the context of exact categories. Recently, a new subclass of these modules called projectively Wakamatsu tilting modules has been introduced. In this talk, we will explain the basics of these modules and present some results: when the algebra is representation-finite, then (projectively) Wakamatsu tilting modules are precisely maximal orthogonal modules, and every orthogonal module can be completed into some Wakamatsu tilting module.

Jun Hu (Beijing Institute of Technology)

Tilting modules, dominant dimension and double centralizer properties

Abstract:
Double centralizer properties play a central role in many parts of algebraic Lie theory and representation theory. For example, in classical Schur-Weyl dualities and Soergel's $\mathbb{V}$ functor on principal block of BGG category $\mathcal{O}$. By the pioneer work of Koenig, Slungard and Xi, we understood that these properties are equivalent to the properties of having dominant dimension at least two with respect to a projective-injective module or a tilting module. In this lecture we shall focus on the question of determining whether a quasi-hereditary algebra A has dominant dimension at least two with respect to a given tilting module T. For any faithful tilting module T over a quasi-hereditary algebra A with a simple-preserving duality, we shall give an affirmative answer to this question, as well as to Mazorchuk and Stroppel's open question on the existence of a unique minimal basic tilting module for which A has double centralizer property.

René Marczinzik (University of Bonn)

On weakly Gorenstein algebras

Abstract:
We show that a large class of algebras, containing monomial algebras and endomorphism rings of modules over representation-finite algebras, are weakly Gorenstein. This generalises a result of Ringel and Zhang. We show new connections between the dominant dimension and semi-Gorenstein projective modules and use this to give a new construction of non-weakly Gorenstein algebras using the theory of gendo-symmetric algebras. If time permits we will discuss some other exotic algebras in representation theory related to the Tachikawa conjectures.

Tiago Cruz (Max Planck Institute for Mathematics, Bonn)

Karin Erdmann (University of Oxford)

Relative dominant dimension with respect to a module

Abstract:

A quasi-hereditary cover of an algebra is a type of resolution of an algebra of possibly infinite global dimension by an algebra of finite global dimension, which has nice properties, in particular it has a natural highest weight structure. The quality of the cover can be measured via relative dominant and codominant dimensions with respect to a summand of a characteristic tilting module.

Part I. In this talk, we discuss the new concept of relative (co-)dominant dimension with respect to a module. It generalises both the classical (co-)dominant dimension and the (co-)faithful dimension in the sense of Buan and Solberg. It is different from the concept used by Koenig, Slungård and Xi but their techniques and applications are compatible with our concept.

Many quasi-hereditary algebras arising in Lie theory possess a simple preserving duality. In this first part, we will focus on the fact that the relative dominant dimension of a quasi-hereditary algebra with a simple preserving duality with respect to a summand Q of the characteristic tilting module is always even. In particular, it is exactly twice the relative dominant dimension of the characteristic tilting module with respect to Q.

Part II. These results are applied to Schur algebras $S_K(2,d)$ when $Q = V^{\otimes d}$, the d-tensor power of the 2-dimensional module. We completely determine the relative dominant dimension of the algebra with respect to Q, for this, the main result of part I is essential. We also obtain the q-analogues. This gives rise to quasi-hereditary covers of Temperley-Lieb algebras. In particular, a byproduct is a Hemmer-Nakano type result connecting the Ringel duals of q-Schur algebras and Temperley-Lieb algebras. All this is compatible with the integral setup.