# Online Satellite Event to
# Homological Methods in Representation Theory

A conference in honour of Lidia Angeleri Hügel

## Abstracts

**Flávio Coelho** (University of São Paulo)
*On generalized path algebras*

The underlined idea behind the concept of generalized path algebras, as introduced in [3], is the following. Given a quiver $Q$ and a field $k$, assign to each of its vertex $x$ a finite dimensional $k$-algebra $A_x$ (in the classical case of path algebra, it is assigned the base field $k$). Then the multiplication is induced by composition of paths and multiplication inside the algebras $A_x$. Also, as in the classical case, we can consider relations in the quiver and define the so-called generalized bound path algebras. An algebra $A = kQ_A/I_A$ (where $Q_A$ is a quiver and $I_A$ is an admissible ideal of $kQ_A$) can naturally be seen as a generalized bound path algebras in two different ways. For one hand, using the quiver $Q_A$ and the usual construction of path algebras, and, on the other, using a quiver with a sole vertex and no arrows and the algebra itself assigned to it. We shall call these ways of representing $A$ as generalized bound path algebras as **trivial**. A question which naturally arises is on the possibility of having descriptions other than the above ones for $A$. Clearly, if this is possible, then to $A$ we shall assign a quiver (generally smaller than its Gabriel quiver) and a set of algebras, one for each vertex of this new quiver, and this might, in principle, allow us to better understand the original algebra. This will be the first question discussed in this talk. Also, we shall discuss the representations of a generalized bound path algebra in terms of the representations of the algebras used in its constructions. Finally, from the description of projective and injective modules, we shall look at some of its homological invariants. We base our talk on a joint work with Viktor Chust [1, 2].

## References

[1] Chust, V., Coelho, F. U., *On the correspondence between path algebras and generalized path algebras*, Communications in Algebra, to appear. arXIv: 2112.12189.

[2] Chust, V., Coelho, F. U., *Representations and Homological Invariants of Generalized Bound Path Algebras*, preprint, arXiv: 2112.12174.

[3] Coelho, F. U., Liu, S.X, *Generalized path algebras in:* Interaction between ring theory and representations of algebras. Proceedings of the conference held in Murcia, Spain. 53–66, Lecture Notes in Pure and Appl. Math., **210**, Dekker, New York, 2000.

**Alberto Facchini** (University of Padova)
*Multiplicative lattices, groups, braces*

The multiplicative lattices we will consider are those defined in [Facchini, Finocchiaro and Janelidze, Abstractly constructed prime spectra, Algebra universalis 83(1) (February 2022)]. Multiplicative lattices yield the natural setting in which several basic mathematical questions concerning algebraic structures find their answer. We will consider the particular cases of groups [Facchini, de Giovanni and Trombetti, Spectra of groups, to appear, 2021] and braces [Facchini, Algebraic structures from the point of view of complete multiplicative lattice, to appear, 2022, http://arxiv.org/abs/2201.03295].

**Birge Huisgen-Zimmermann** (University of California, Santa Barbara)
*Delooping levels versus finitistic dimensions*

Recently, Vincent Gélinas introduced a new homological invariant of a noetherian ring, which he named the delooping level. Its strong relationship to depth and finitistic dimensions makes this invariant a promising tool towards a better understanding of the latter. We will provide examples, review those of Gelinas's results which pertain to Artin algebras, and answer the concluding question of his initial paper in the negative. Moreover, we will modestly enlarge the class of algebras for which the delooping level, and hence also the big finitistic dimension, is known to be finite.

**Tsutomu Nakamura** (University of Tokyo)
*Indecomposable pure-injective objects in stable categories of Gorenstein-projective modules over Gorenstein orders*

I will give a result of Auslander-Ringel-Tachikawa type for Gorenstein-projective modules over a complete Gorenstein order. In particular, it will be shown that a complete Gorenstein order is of finite Cohen-Macaulay type if and only if every indecomposable pure-injective object in the stable category of Gorenstein-projective modules is compact. One of key results to this aim is due to Rosanna Laking.

**Javier Sanchez** (University of São Paulo)
*On graded division rings*

Given an associative ring with unit, P. M. Cohn characterized the homomorphisms from $R$ to division rings by means of a structure defined over the set of square matrices over $R$. P. Malcolmson described alternative ways of determining such homomorphisms using functions induced from the notions of rank of a matrix and of dimension of modules over division rings.

In this work, we show that these characterizations can be implemented in the context of graded rings. More precisely, given a ring $R$ graded by a group $G$ we adapt the theory of Cohn and Malcolmson to determine the different graded homomorphisms from $R$ to $G$-graded division rings.

This is a joint work with Daniel E. N. Kawai.

**Simone Virili** (University of Udine)
*Length functions on Grothendieck categories*

Given a Grothendieck category $\mathfrak{G}$, a *length function* $L\colon \mathfrak{G} \to \mathbb{R}_{\geq 0} \cup \{\infty\}$ is a non-negative real invariant (that may attain infinity) of the objects of $\mathfrak{G}$ with two essential properties: it is *additive* on short exact sequences and it is *continuous* on direct unions. Classical examples of length functions are the dimension of vector spaces, the rank of modules over Ore domains, or the composition length, but also genuinely non-discrete examples exist. The problem of classification of all the length functions on a given Grothendieck category $\mathfrak{G}$ seems hopeless in general but, in some cases, one can write all the length functions on $\mathfrak{G}$ as linear combinations of a family of *indecomposable length functions*. One such example is the following: if $\mathfrak{G}$ has Gabriel dimension (e.g., $\mathfrak{G}$ is locally Noetherian) then the (equivalence classes of) indecomposable length functions are in bijection with the (isomorphism classes of) indecomposable injective objects, and every other length function is a linear combination of those. One can even go one step further and prove that, in general, the equivalence classes of *discrete* (i.e., taking values in a subset of $\mathbb{R}_{\geq 0} \cup \{\infty\}$ which is order isomorphic to $\mathbb{N} \cup \{\infty\}$) indecomposable length functions are in bijection with the so-called *atom spectrum* of $\mathfrak{G}$. Hence, the family of all the indecomposable length functions (so including also the non-discrete ones) can be considered as an extension of this spectrum. In this talk, I will summarize these classification results and I will discuss some open problems and possible future directions of this investigation.