Quiver Schur algebras

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Outline

1. Motivation
2. Cyclotomic Hecke algebras and quiver Hecke algebras
3. An x-deformation of the quiver Hecke algebras of type A
4. Cyclotomic q-Schur and quiver Schur algebras
5. An x-deformation of the quiver Schur algebras

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Type A Iwahori–Hecke algebras at roots of unity

Notations

$n \in \mathbb{N}$, e is an integer such that $e \geq 2$;

K is the ground field;

$\xi \in K^\times$ such that e is the minimal positive integer satisfying $1 + \xi + \xi^2 + \cdots + \xi^{e-1} = 0$;

$\mathcal{H}_\xi(\mathfrak{S}_n)$ is the Iwahori–Hecke algebra (over K) associated to the symmetric group $\mathfrak{S}_n$ with parameter ξ;

$\mathcal{P}_n$ is the set of partitions of n.

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Specht modules, simple modules and decomposition numbers

S_K^λ: the Specht module corresponding to $\lambda \in \mathcal{P}_n$;

D_K^μ: the simple module corresponding to $\mu \in \mathcal{P}_n$ with μ being e-restricted (i.e., $\mu_i - \mu_{i+1} < e$ for all i);

$d_{\lambda,\mu}^K := [S_K^\lambda : D_K^\mu] \in \mathbb{Z}^{\geq 0}$: the decomposition number (i.e., the multiplicity of D_K^μ as a composition factor in S_K^λ).

Problem A: Compute (or find a simple algorithm to compute) $\dim D_K^\mu$ and $d_{\lambda,\mu}^K$.

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Independent of K or not?

In the case char $K = 0$, there is a solution to Problem A(due to Ariki–Leclerc–Thibon–Lascoux) which uses the theory of Fock spaces, canonical bases and affine parabolic Kazhdan–Lusztig polynomials.

In general, the problem remains open.

James's Conjecture (James, Geck, Fayers, $\cdots$)

Let $p :=$ char K. If $pe > n$ or $p > w$, where w is the e-weight of μ, then $d^K_{\lambda,\mu} = d^{\mathbb{C}}_{\lambda,\mu}$ and $\dim D^\mu_K = \dim D^\mu_{\mathbb{C}}$.

There is a similar q-Schur algebra version of the above conjecture.

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Cyclotomic Hecke algebras of type $G(\ell, 1, n)$

The cyclotomic Hecke algebras $\mathcal{H}_n(q, \mathbf{Q})$ of type $G(\ell, 1, n)$ are some generalisations of the type A Iwahori–Hecke algebras $\mathcal{H}_\xi(\mathfrak{S}_n)$ and the type B Iwahori–Hecke algebras.

$q \in K^\times$, the Hecke parameter;

$(\kappa_1, \ldots, \kappa_\ell) \in \mathbb{Z}^\ell$: the multi-charge.

$\mathbf{Q} = (q_{\kappa_1}, \ldots, q_{\kappa_\ell})$: the cyclotomic parameters,

where for an integer $a \in \mathbb{Z}$ we define

$$q_a = \begin{cases} q^a, & \text{if } q \neq 1, \\ a, & \text{if } q = 1. \end{cases}$$

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Cyclotomic Hecke algebras of type $G(\ell, 1, n)$

- If $q \neq 1$, then $\mathcal{H}_n(q, \mathbf{Q})$ is isomorphic to the non degenerated cyclotomic Hecke algebra of type $G(\ell, 1, n)$;
- If $q = 1$, then $\mathcal{H}_n(q, \mathbf{Q})$ is isomorphic to the degenerate cyclotomic Hecke algebra of type $G(\ell, 1, n)$.

Let e be the minimal positive integer such that $1 + q + q^2 + \cdots + q^{e-1} = 0$;

The representation theory of $\mathcal{H}_n(q, \mathbf{Q})$ is very similar to that of $\mathcal{H}_\xi(\mathfrak{S}_n)$.

Problem B: To which extent the representation theory of $\mathcal{H}_n(q, \mathbf{Q})$ depends only on e but not on the characteristic of the ground field K?.

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Problem B: To which extent the representation theory of $\mathcal{H}_n(q, \mathbf{Q})$ depends only on e but not on the characteristic of the ground field K?.

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- If $q \neq 1$, then $\mathcal{H}_n(q, \mathbf{Q})$ is isomorphic to the non degenerated cyclotomic Hecke algebra of type $G(\ell, 1, n)$;
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Quiver Hecke algebras of type A

In 2008, Khovanov and Lauda, and Rouquier have introduced a remarkable family of $\mathbb{Z}$-graded algebras which are now known to categorify the canonical bases of Kac-Moody algebras.

Brundan and Kleshchev initiated the study of cyclotomic quotients of these algebras by showing that in the case of type A they are isomorphic to the degenerate and non-degenerate cyclotomic Hecke algebras of type $G(\ell, 1, n)$.

In this talk, I will concentrated on the type A and linear quiver case. We assume that $e = 0$ and q is not a root of unity.

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Quiver Hecke algebras of type A when $e = 0$

Definition (Khovanov-Lauda, Rouquier)

Let $\alpha \in Q_n^+$ and R_α be the unital associative K-algebra with generators: $\{\psi_1, \ldots, \psi_{n-1}\} \cup \{y_1, \ldots, y_n\} \cup \{e(\mathbf{i}) | \mathbf{i} \in I^\alpha\}$ and relations

$$e(\mathbf{i})e(\mathbf{j}) = \delta_{\mathbf{ij}} e(\mathbf{i}), \quad \sum_{\mathbf{i} \in I^\alpha} e(\mathbf{i}) = 1,$$
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$$\psi_r^2 e(\mathbf{i}) = \begin{cases} 0, & \text{if } i_r = i_{r+1}, \\ e(\mathbf{i}), & \text{if } i_r \neq i_{r+1} \pm 1, \\ (y_{r+1} - y_r)e(\mathbf{i}), & \text{if } i_{r+1} = i_r + 1, \\ (y_r - y_{r+1})e(\mathbf{i}), & \text{if } i_{r+1} = i_r - 1, \end{cases}$$

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for $\mathbf{i}, \mathbf{j} \in I^\alpha$ and all admissible r, s.

Quiver Hecke algebras of type A

- The algebra R_α is called Khovanov-Lauda-Rouquier algebra of type A, or quiver Hecke algebra of type A, associated to the linear quiver;

Corollary

There is a unique $\mathbb{Z}$-grading on R_α such that $e(\mathbf{i})$ is of degree 0, y_r is of degree 2, and $\psi_r e(\mathbf{i})$ is of degree $-a_{i_r,i_{r+1}}$ for each r and $i \in I^\alpha$.

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Let $\Lambda \in P_+$ be a fixed dominant weight.

Definition

The cyclotomic Khovanov-Lauda-Rouquier algebras associated to the weight Λ and $\alpha \in Q_+$ is the quotient algebra of R_α by the two-sided ideal generated by $y_1^{(\Lambda, \alpha_{i_1})} e(\mathbf{i})$, for all $\mathbf{i} \in I^\alpha$.

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$$I^\alpha := \{\mathbf{i} = (i_1, \cdots, i_n) \in \mathbb{Z}^n | \alpha_{i_1} + \cdots + \alpha_{i_n} = \alpha\}.$$

We define

$$\Lambda := \Lambda_{\kappa_1} + \cdots + \Lambda_{\kappa_\ell},$$
$$H_n^\Lambda := \mathcal{H}_n(q, \mathbf{Q}).$$

Recall that $e(\alpha) = \sum_{\mathbf{i} \in I^\alpha} e(\mathbf{i})$.

Assume that $e(\alpha) \neq 0$. That says, there exists $\mathfrak{t} \in \mathrm{Std}(\lambda)$ for some multipartition λ, such that $\mathbf{i}^{\mathfrak{t}} \in I^\alpha$. We define

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Theorem (Brundan-Kleshchev, Invent. Math., 178, (2009))

There is a K-algebra isomorphism $R_\alpha^\Lambda \cong H_\alpha^\Lambda$.

Note that

$$H_n^\Lambda = \oplus_{\alpha\in Q_n^+} e(\alpha) H_n^\Lambda = \oplus_{\alpha\in Q_n^+} H_\alpha^\Lambda.$$

Henceforth, We shall use Brundan-Kleshchev's isomorphism to identify H_α^Λ with R_α^Λ for each $\alpha \in Q_n^+$.

In particular, each block algebra H_α^Λ was endowed with a nontrivial $\mathbb{Z}$-grading, and the cyclotomic Hecke algebra H_n^Λ becomes a $\mathbb{Z}$-graded algebra.

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Some combinatorics

$\mathcal{P}_n$: the set of multipartitions $\lambda = (\lambda^{(1)}, \cdots, \lambda^{(\ell)})$ of n;

$\mathcal{P}_n$ is partially ordered by the dominance order "$\trianglerighteq$", where $\lambda \trianglerighteq \mu$ if

$$\sum_{t=1}^{s-1} |\lambda^{(t)}| + \sum_{i=1}^{j} \lambda_i^{(s)} \geqslant \sum_{t=1}^{s-1} |\mu^{(t)}| + \sum_{i=1}^{j} \mu_i^{(s)}$$

for all $1 \leq s \leq \ell$ and all $j \geqslant 1$;

For $\lambda \in \mathcal{P}_n$, the Young diagram $[\lambda]$ is defined to be the set

$$[\lambda] := \{(r, c, l) | 1 \leq c \leq \lambda_r^{(l)}, 1 \leq l \leq \ell\}.$$

A λ-tableau $\mathfrak{t}$ is a bijective map $\mathfrak{t} : [\lambda] \to \{1, 2, \ldots, n\}$. We can write $\mathfrak{t} = (\mathfrak{t}^{(1)}, \cdots, \mathfrak{t}^{(\ell)})$.

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Some combinatorics

$\mathcal{P}_n$: the set of multipartitions $\lambda = (\lambda^{(1)}, \cdots, \lambda^{(\ell)})$ of n;

$\mathcal{P}_n$ is partially ordered by the dominance order "$\trianglerighteq$", where $\lambda \trianglerighteq \mu$ if

$$\sum_{t=1}^{s-1} |\lambda^{(t)}| + \sum_{i=1}^{j} \lambda_i^{(s)} \geqslant \sum_{t=1}^{s-1} |\mu^{(t)}| + \sum_{i=1}^{j} \mu_i^{(s)}$$

for all $1 \leq s \leq \ell$ and all $j \geqslant 1$;

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$\mathrm{Std}(\lambda)$: the set of standard λ-tableaux.

For $\mathfrak{s} \in \mathrm{Std}(\lambda)$, let $\mathfrak{s}_k$ be the subtableau of $\mathfrak{s}$ labelled by $1, 2, \cdots, k$;

Given $\mathfrak{s} \in \mathrm{Std}(\lambda)$, $\mathfrak{t} \in \mathrm{Std}(\mu)$, we write $\mathfrak{s} \trianglerighteq \mathfrak{t}$ if $\mathrm{Shape}(\mathfrak{s}_k) \trianglerighteq \mathrm{Shape}(\mathfrak{t}_k)$, for $k = 1, \ldots, n$.

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Degree of tableaux

Let $\lambda \in \mathcal{P}_n$ and $\mathfrak{t} \in \mathrm{Std}(\lambda)$. For $k = 1, \dots, n$ let $A_{\mathfrak{t}}(k)$ be the set of addable nodes of the multipartition $\mathrm{Shape}(\mathfrak{t}_k)$ which are *below* $\mathfrak{t}^{-1}(k)$. Let $R_{\mathfrak{t}}(k)$ be the set of removable nodes of $\mathrm{Shape}(\mathfrak{t}_k)$ which are *below* $\mathfrak{t}^{-1}(k)$.

Definition (Brundan-Kleshchev-Wang)

We define

$$A_{\mathfrak{t}}^{\Lambda}(k) = \{\alpha \in A_{\mathfrak{t}}(k) \mid \mathrm{res}(\alpha) = \mathrm{res}_{\mathfrak{t}}(k)\},$$

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Ungraded cellular basis

Recall that (Dipper-James-Mathas) the cyclotomic Hecke algebra H_n^Λ is a cellular algebra with cellular basis

$$\{m_{\mathfrak{s},\mathfrak{t}} | \mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda), \lambda = (\lambda^{(1)}, \cdots, \lambda^{(\ell)}) \vdash n\}.$$

The corresponding cell module (i.e., Specht module) S_λ has a natural basis of the form

$$\{m^\lambda T_{d(\mathfrak{t})} + H_n^{\rhd\lambda} | \mathfrak{t} \in \mathrm{Std}(\lambda)\},$$

where $d(\mathfrak{t}) \in \mathfrak{S}_n$ such that $\mathfrak{t}^\lambda d(\mathfrak{t}) = \mathfrak{t}$, $\mathfrak{t}^\lambda$ is the initial standard λ-tableau.

However, both of these bases are in general not homogeneous.

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Graded cellular algebras

Let R be a $\mathbb{Z}$-graded commutative integral domain.

Definition (Graded cellular algebras)

Suppose that A is a $\mathbb{Z}$-graded R-algebra which is free of finite rank over R. A **graded cell datum** for A is an ordered quadruple $(\mathcal{P}, T, C, \deg)$, where $(\mathcal{P}, \rhd)$ is the **weight poset**, $T(\lambda)$ is a finite set for $\lambda \in \mathcal{P}$, and

$$C \to \coprod_{\lambda\in\mathcal{P}} T(\lambda)\times T(\lambda) \to A; (\mathfrak{s},\mathfrak{t}) \mapsto c^{\lambda}_{\mathfrak{s},\mathfrak{t}}, \text{ and } \deg : \coprod_{\lambda\in\mathcal{P}} T(\lambda) \to \mathbb{Z}$$

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are two functions such that C is injective and

(GC1) $\{c^{\lambda}_{\mathfrak{s},\mathfrak{t}} \mid \mathfrak{s},\mathfrak{t} \in T(\lambda), \lambda \in \mathcal{P}\}$ is an R-basis of A.

(GC2) Each basis element $c^{\lambda}_{\mathfrak{s},\mathfrak{t}}$ is homogeneous of degree $\deg c^{\lambda}_{\mathfrak{s},\mathfrak{t}} = \deg \mathfrak{s} + \deg \mathfrak{t}$, for $\lambda \in \mathcal{P}$ and $\mathfrak{s},\mathfrak{t} \in T(\lambda)$.

(GC3) If $\mathfrak{s},\mathfrak{t} \in T(\lambda)$, for some $\lambda \in \mathcal{P}$, and $a \in A$ then there exist scalars $r_{\mathfrak{t},\mathfrak{v}}(a)$, which do not depend on $\mathfrak{s}$, such that

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A **graded cellular algebra** is a graded R-algebra which has a graded cell datum. The basis $\{c^{\lambda}_{\mathfrak{s},\mathfrak{t}} | \lambda \in \mathcal{P} \text{ and } \mathfrak{s},\mathfrak{t} \in T(\lambda)\}$ is a **graded cellular basis** of A.

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Definition

Let $\lambda \in \mathcal{P}_n$. We define

$$y^{\lambda} := \prod_{k=1}^{n} y_k^{|A_{\mathfrak{t}^\lambda}^{\Lambda}(k)|}.$$

In particular, $\deg(y^{\lambda}) = 2\deg(\mathfrak{t}^{\lambda})$.

For each $\mathfrak{t} \in \mathrm{Std}(\lambda)$, we fix a reduced expression $s_{j_1} s_{j_2} \cdots s_{j_k}$ of $d(\mathfrak{t})$ and define $\psi_{\mathfrak{t}} = \psi_{j_1}\psi_{j_2}\cdots\psi_{j_k}$.

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Graded cellular bases

Recall that H_n^Λ has a unique K-linear anti-automorphism "$*$" which fixes each of the graded generators.

Theorem (Hu-Mathas)

For each $\lambda \in \mathcal{P}_n$ and $\mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda)$, we define

$$\psi_{\mathfrak{s},\mathfrak{t}} := \psi_{\mathfrak{s}}^* e^\lambda y^\lambda \psi_{\mathfrak{t}}.$$

The algebra H_n^Λ is a graded cellular algebra with weight poset $(\mathcal{P}_n, \trianglerighteq)$ and graded cellular basis $\{\psi_{\mathfrak{s},\mathfrak{t}} | \mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda) \text{ for } \lambda \in \mathcal{P}_n\}$. In particular, $\deg(\psi_{\mathfrak{s},\mathfrak{t}}) = \deg \mathfrak{s} + \deg \mathfrak{t}$, for all $\mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda)$, $\lambda \in \mathcal{P}_n$.

Graded cellular bases

Recall that H_n^Λ has a unique K-linear anti-automorphism "$*$" which fixes each of the graded generators.

Theorem (Hu-Mathas)

For each $\lambda \in \mathcal{P}_n$ *and* $\mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda)$, *we define*

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Graded cellular bases

The relation between the graded cellular basis $\psi_{\mathfrak{s},\mathfrak{t}}$ and the ungraded cellular basis $m_{\mathfrak{s},\mathfrak{t}}$ is given by the following result.

Lemma

Let $\lambda \in \mathcal{P}_n$ and $\mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda)$. Then there exists a non-zero scalar $c \in K$, such that

$$\psi_{\mathfrak{s},\mathfrak{t}} = c m_{\mathfrak{s},\mathfrak{t}} + \sum_{(\mathfrak{u},\mathfrak{v}) \rhd (\mathfrak{s},\mathfrak{t})} r_{\mathfrak{u},\mathfrak{v}} m_{\mathfrak{u},\mathfrak{v}},$$

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Homogeneous trace form

Definition (Homogeneous trace)

Suppose that $\alpha \in Q_+$. Then $\tau_\alpha : H^\Lambda_\alpha \to K$ is the map which on a homogeneous element $a \in H^\Lambda_\alpha$ is given by

$$\tau_\alpha(a) = \begin{cases} \tau(a), & \text{if } \deg(a) = 2\operatorname{def}\alpha, \\ 0, & \text{otherwise.} \end{cases}$$

We define a homogeneous bilinear form $\langle\ ,\ \rangle_\alpha$ on H^Λ_α of degree $-2\operatorname{def}\alpha$ by $\langle a, b\rangle_\alpha = \tau_\alpha(ab^*)$.

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In Andrew's talk, for each e-idempotent local subring R, we can give a KLR-like presentation for the cyclotomic Hecke algebra defined $\mathcal{H}_{n,\mathcal{O}}^{\Lambda}$ over $\mathcal{O}$. We shall call $\mathcal{H}_{n,\mathcal{O}}^{\Lambda}$ an x-deformation of the cyclotomic quiver Hecke algebra of type A.

We already noticed KLR-like presentation for $\mathcal{H}_{n,\mathcal{O}}^{\Lambda}$ is not always homogeneous in an obvious way. However, for certain choices of the e-idempotent local subring, we can make the presentation homogeneous.

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Some notations

Let $q, x^{\mathbf{Q}_1}, x^{\mathbf{D}_1}, x^{\mathbf{D}_2}, \cdots, x^{\mathbf{D}_{\ell-1}}$ be $\ell+1$ commuting indeterminates over $\mathbb{Z}$. We define

$$\mathcal{A} := \mathbb{Q}(q)[x^{\mathbf{Q}_1}, x^{\mathbf{D}_1}, \cdots, x^{\mathbf{D}_{\ell-1}}],$$
$$\mathcal{A}_0 := \mathbb{Q}(q)[x^{\mathbf{Q}_1}, x^{\mathbf{D}_1}, \cdots, x^{\mathbf{D}_{\ell-1}}]_{(x^{\mathbf{Q}_1}, x^{\mathbf{D}_1}, \cdots, x^{\mathbf{D}_{\ell-1}})},$$
$$\mathcal{K} := \mathbb{Q}(x^{\mathbf{Q}_1}, x^{\mathbf{D}_1}, \cdots, x^{\mathbf{D}_{\ell-1}}, q).$$

We regard $\mathcal{A}$ as a $\mathbb{Z}$-graded algebra by putting

$$\deg x^{\mathbf{Q}_1} := 2, \ \deg x^{\mathbf{D}_i} := 0, \ \forall\, 1 \le i \le \ell-1,$$
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$\mathfrak{m}_0$-adic completion

Let $\mathfrak{m}$ be the maximal ideal of $\mathcal{A}$ generated by $x^{\mathbf{Q}_1}, x^{\mathbf{D}_1}, \cdots, x^{\mathbf{D}_{\ell-1}}$. Let $\mathfrak{m}_0$ be the unique maximal ideals $\mathcal{A}_0$.

We use $\widehat{\mathcal{A}}_0$ to denote the corresponding $\mathfrak{m}_0$-adic completion of $\mathcal{A}_0$. By definition,

$$\widehat{\mathcal{A}}_0 = \varprojlim_k \mathcal{A}_0/\mathfrak{m}_0^k \cong \varprojlim_{k\geq 0} \mathcal{A}/\mathfrak{m}^k.$$

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An x-deformation of the quiver Hecke algebras of type A

We consider the cyclotomic Hecke algebras $\mathcal{H}_{n,\mathcal{A}_0}$ and $\mathcal{H}_{n,\mathcal{K}}$ with parameters q; $x^{\mathbf{Q}_1} + q^{\kappa_1}, x^{\mathbf{Q}_1+\mathbf{D}_1} + q^{\kappa_1}, \cdots, x^{\mathbf{Q}_1+\mathbf{D}_{\ell-1}} + q^{\kappa_\ell}$.

Theorem (Hu-Mathas)

the algebra $\mathcal{H}_{n,\mathcal{A}_0}$ is isomorphic to the $\mathcal{A}_0$-algebra which has a presentation given by the generators:

$$\left\{f_{\mathbf{i}}^{\mathcal{A}_0} \mid \mathbf{i} \in I^n\right\} \cup \left\{\psi_r^{\mathcal{A}_0} \mid 1 \le r < n\right\} \cup \left\{y_r^{\mathcal{A}_0} \mid 1 \le r \le n\right\}, \qquad (1)$$

and the following relations:

$$\prod_{\substack{1 \le l \le \ell \\ i_1 = \kappa_l}} (q^{i_1} y_1^{\mathcal{A}_0} - x^{\mathbf{Q}_1+\mathbf{D}_{l-1}}) f_{\mathbf{i}}^{\mathcal{A}_0} = 0,$$

Generators and relations (continued)

$$f_{\mathbf{i}}^{\mathcal{A}_0} f_{\mathbf{j}}^{\mathcal{A}_0} = \delta_{\mathbf{ij}} f_{\mathbf{i}}^{\mathcal{A}_0}, \quad \sum_{\mathbf{i}\in I^n} f_{\mathbf{i}}^{\mathcal{A}_0} = 1,$$

$$y_r^{\mathcal{A}_0} f_{\mathbf{i}}^{\mathcal{A}_0} = f_{\mathbf{i}}^{\mathcal{A}_0} y_r^{\mathcal{A}_0}, \quad \psi_r^{\mathcal{A}_0} f_{\mathbf{i}}^{\mathcal{A}_0} = f_{s_r\mathbf{i}}^{\mathcal{A}_0} \psi_r^{\mathcal{A}_0}, \quad y_r^{\mathcal{A}_0} y_s^{\mathcal{A}_0} = y_s^{\mathcal{A}_0} y_r^{\mathcal{A}_0},$$

$$\psi_r^{\mathcal{A}_0} y_{r+1}^{\mathcal{A}_0} f_{\mathbf{i}}^{\mathcal{A}_0} = (y_r^{\mathcal{A}_0}\psi_r^{\mathcal{A}_0} + \delta_{i_r i_{r+1}}) f_{\mathbf{i}}^{\mathcal{A}_0}, y_{r+1}^{\mathcal{A}_0}\psi_r^{\mathcal{A}_0} f_{\mathbf{i}}^{\mathcal{A}_0} = (\psi_r^{\mathcal{A}_0} y_r^{\mathcal{A}_0} + \delta_{i_r i_{r+1}}) f_{\mathbf{i}}^{\mathcal{A}}$$

$$\psi_r^{\mathcal{A}_0} y_s^{\mathcal{A}_0} = y_s^{\mathcal{A}_0}\psi_r^{\mathcal{A}_0}, \qquad \text{if } s \neq r, r+1,$$

$$\psi_r^{\mathcal{A}_0}\psi_s^{\mathcal{A}_0} = \psi_s^{\mathcal{A}_0}\psi_r^{\mathcal{A}_0}, \qquad \text{if } |r-s| > 1,$$

$$(\psi_r^{\mathcal{A}_0})^2 f_{\mathbf{i}}^{\mathcal{A}_0} = \begin{cases} (y_r^{\mathcal{A}_0} - y_{r+1}^{\mathcal{A}_0}) f_{\mathbf{i}}^{\mathcal{A}_0}, & \text{if } i_r \rightarrow i_{r+1}, \\ (y_{r+1}^{\mathcal{A}_0} - y_r^{\mathcal{A}_0}) f_{\mathbf{i}}^{\mathcal{A}_0}, & \text{if } i_r \leftarrow i_{r+1}, \\ 0, & \text{if } i_r = i_{r+1}, \\ f_{\mathbf{i}}^{\mathcal{A}_0}, & \text{otherwise.} \end{cases}$$

Theorem (continued)

Moreover, the isomorphism sends $(q^{-i_k}L_k - 1)f_{\mathbf{i}}^{\mathcal{A}_0}$ to $y_k^{\mathcal{A}_0} f_{\mathbf{i}}^{\mathcal{A}_0}$ for each $\mathbf{i} \in I^n$ and $1 \le k \le n$. Similar results are also true if we replace $\mathcal{A}_0$ by $\mathcal{K}$.

Let "$\star$" be the anti-involution which is identity on the generators (1).

Definition

For each $\lambda \in \mathcal{P}_n$, we define

$$y_{\mathcal{A}_0}^{\lambda} := \prod_{k=1}^{n} \prod_{\alpha=(l,r,c)\in\mathscr{A}_{\mathfrak{t}^\lambda}(k)} ((x^{\mathbf{Q}_1+\mathbf{D}_{l-1}} + q^{\kappa_l})q^{c-r} - L_k), \quad e_{\mathcal{A}_0}^{\lambda} := f_{\mathbf{i}^{\mathfrak{t}^\lambda}}^{\mathcal{A}_0}.$$

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For any $\mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda)$, we define

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Similarly, we have the corresponding definitions with $\mathcal{A}_0$ replaced by $\mathcal{K}$.

We define

$$\deg(f_{\mathbf{i}}^{\mathcal{A}_0}) = 0, \quad \deg(y_r^{\mathcal{A}_0}) = 2, \quad \deg(\psi_r^{\mathcal{A}_0} f_{\mathbf{i}}^{\mathcal{A}_0}) = -a_{i_r, i_{r+1}},$$

where $(a_{i,j})$ is the Cartan matrix associated to the linear quiver. With these definitions, we remark that all the relations appeared in the previous lemma are homogeneous.

Homogeneous cellular basis

Lemma

Let $\lambda \in \mathcal{P}_n$ *and* $\mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda)$. *Then* $\psi_{\mathfrak{s},\mathfrak{t}}^{\mathcal{A}_0}$ *is a homogeneous element of degree* $\deg \mathfrak{s} + \deg \mathfrak{t}$. *Furthermore, The set* $\{\psi_{\mathfrak{s},\mathfrak{t}}^{\mathcal{A}_0} \mid \mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda), \lambda \in \mathcal{P}_n\}$ *is a cellular* $\mathcal{A}_0$*-basis of* $\mathcal{H}_{n,\mathcal{A}_0}$.

The above lemma is also true if we replace $\mathcal{A}_0$ by $\widehat{\mathcal{A}_0}$.

As a result, we can regard $\mathcal{H}_{n,\widehat{\mathcal{A}_0}}^{\Lambda}$ as a generalised $\mathbb{Z}$-graded cellular $\widehat{\mathcal{A}_0}$-algebras.

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As a result, we can regard $\mathcal{H}^{\Lambda}_{n,\widehat{\mathcal{A}_0}}$ as a generalised $\mathbb{Z}$-graded cellular $\widehat{\mathcal{A}_0}$-algebras.

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Remark

Let x be an indeterminant over $\mathbb{Z}$. Assume that $Q_1, \cdots, Q_l$ are pairwise distinct positive integers. We define $\mathcal{O}_0 := \mathbb{Q}[x, q]_{(x)}$ and $K := \mathbb{Q}(x, q)$. Let $R \in \{\mathcal{O}_0, K\}$. We can also consider the cyclotomic Hecke algebras $\mathcal{H}_{n,R}$ with parameters

$$q; x^{Q_1} + q^{\kappa_1}, \cdots, x^{Q_\ell} + q^{\kappa_\ell}.$$

It is clear that the algebra $\mathcal{H}_{n,K} := \mathcal{H}_{n,\mathcal{O}} \otimes_{\mathcal{O}_0} K$ is semisimple.

Corollary

All the previous results (except those involve homogeneous) are true if we replace $\mathcal{A}_0$ and $\mathcal{K}$ by $\mathcal{O}_0$ and K respectively.

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Let x be an indeterminant over $\mathbb{Z}$. Assume that $Q_1, \cdots, Q_l$ are pairwise distinct positive integers. We define $\mathcal{O}_0 := \mathbb{Q}[x, q]_{(x)}$ and $K := \mathbb{Q}(x, q)$. Let $R \in \{\mathcal{O}_0, K\}$. We can also consider the cyclotomic Hecke algebras $\mathcal{H}_{n,R}$ with parameters

$$q; x^{Q_1} + q^{\kappa_1}, \cdots, x^{Q_\ell} + q^{\kappa_\ell}.$$

It is clear that the algebra $\mathcal{H}_{n,K} := \mathcal{H}_{n,\mathcal{O}} \otimes_{\mathcal{O}_0} K$ is semisimple.

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All the previous results (except those involve homogeneous) are true if we replace $\mathcal{A}_0$ and $\mathbb{K}$ by $\mathcal{O}_0$ and K respectively.

Homogeneous trace form

Definition

Suppose that $\beta \in Q_n^+$. Let $\tau_\beta^{\widehat{\mathcal{A}}_0} : \mathcal{H}^\Lambda_{\beta,\widehat{\mathcal{A}}_0} \to (\widehat{\mathcal{A}}_0)_0$ be the map which on a homogeneous element $a \in \mathcal{H}^\Lambda_{\beta,\widehat{\mathcal{A}}_0}$ is given by

$$\tau_\beta^{\widehat{\mathcal{A}}_0}(a) = \begin{cases} (\tau(a))_0, & \text{if } \deg(a) = 2\operatorname{def}\beta, \\ 0, & \text{otherwise,} \end{cases}$$

where $(\tau(a))_0$ means the degree 0 component of $\tau(a) \in \widehat{\mathcal{A}}_0$. Let $\tau_\beta^{\widehat{\mathcal{K}}} : \mathcal{H}^\Lambda_{\beta,\widehat{\mathcal{K}}} \to \widehat{\mathcal{K}}$ be the natural $\widehat{\mathcal{K}}$-linear extension of the map $\tau_\beta^{\widehat{\mathcal{A}}_0}$.

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Suppose that $\beta \in Q_n^+$. Then $\tau_\beta^{\widehat{\mathcal{K}}}$ is a non-degenerate "homogeneous" trace form on $\mathcal{H}_{\beta,\widehat{\mathcal{K}}}^\Lambda$ of "degree" $-2\operatorname{def}\beta$.

Lemma

Let $\lambda, \mu \in \mathcal{P}_n$.

$$S_{\widehat{\mathcal{K}}}^\lambda \cong \left(S_\lambda^{\widehat{\mathcal{K}}}\right)^{\circledast}\langle \operatorname{def}\beta\rangle. \tag{2}$$

Furthermore,

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(Ungraded) permutation modules

Let $\lambda \in \mathcal{P}_n^\Lambda$. If $1 \le k \le n$ and $\mathfrak{t} = (\mathfrak{t}^{(1)}, \dots, \mathfrak{t}^{(\ell)})$ is a tableau then $\mathrm{comp}_\mathfrak{t}(k) = s$ if k appears in $\mathfrak{t}^{(s)}$. Define $m^\lambda = u^\lambda x^\lambda$ where

$$u^\lambda = \prod_{k=1}^{n} \prod_{s=\mathrm{comp}_{\mathfrak{t}^\lambda}(k)+1}^{\ell} (L_k - q_{\kappa_s}) \quad \text{and} \quad x^\lambda = \sum_{w \in \mathfrak{S}_\lambda} T_w,$$

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Graded cyclotomic q-Schur algebras

Definition

The **cyclotomic Schur algebra** is the algebra

$$\underline{S}_n^{\mathrm{DJM}} = \mathrm{End}_{H_n^\Lambda}\Big(\bigoplus_{\lambda\in\mathcal{P}_n^\Lambda} \underline{M}^\lambda\Big).$$

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Reference:
Hu–Mathas, Quiver Schur algebras for the linear quivers, I, arXiv:1110.1699, (2011).

Definition

Let $\mu \in \mathcal{P}_n^\Lambda$. We define

$$G^\mu = e^\mu y^\mu R_n^\Lambda \langle -\deg \mathfrak{t}^\mu \rangle.$$

We call G^μ the graded permutation module associated to μ.

If $\mu, \lambda \in \mathcal{P}_n^\Lambda$ define

$$\mathrm{Std}^\mu(\lambda) = \{\mathfrak{s} \in \mathrm{Std}(\lambda) \mid \mathfrak{s} \trianglerighteq \mathfrak{t}^\mu \text{ and } \mathrm{res}(\mathfrak{s}) = \mathbf{i}^\mu\}.$$

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Theorem

Suppose that $\mu \in \mathcal{P}_n^\Lambda$. Then G^μ is a free module with basis given by

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Definition (Hu–Mathas)

Let $G_n^\Lambda = \bigoplus_{\mu\in\mathcal{P}_n^\Lambda} G^\mu$. The quiver Schur algebra of type (Γ_e, Λ) is the endomorphism algebra

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Theorem (Hu–Mathas)

Suppose that $e = 0$ or $e \geqslant n$ and let $\mathcal{Z}$ be an integral domain such that e is invertible in $\mathcal{Z}$ whenever $e \neq 0$ and e is not prime. Then S_n^Λ is a graded cellular algebra with cellular basis

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For $\lambda \in \mathcal{P}_n^\Lambda$, let $\mathcal{T}^\lambda := \{(\mu, \mathfrak{s}) \mid \mathfrak{s} \in \mathrm{Std}^\mu(\lambda) \text{ for } \mu \in \mathcal{P}_n^\Lambda\}$.

Theorem (Hu–Mathas)

Suppose that $e = 0$ or $e \geqslant n$ and let $\mathcal{Z}$ be an integral domain such that e is invertible in $\mathcal{Z}$ whenever $e \neq 0$ and e is not prime. Then $\mathcal{S}_n^\Lambda$ is a graded cellular algebra with cellular basis

$$\{\Psi_{\mathfrak{s}\mathfrak{t}}^{\mu\nu} \mid (\mu, \mathfrak{s}), (\nu, \mathfrak{t}) \in \mathcal{T}^\lambda \text{ and } \lambda \in \mathcal{P}_n^\Lambda,$$

weight poset $(\mathcal{P}_n^\Lambda, \trianglerighteq)$ and degree function

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Corollary

Suppose that $e = 0$ or $e \geqslant n$. Then $\mathcal{S}_n^\Lambda$ is quasi-hereditary graded cellular algebra.

Theorem (Hu–Mathas)

Suppose that $\mathcal{Z}$ is a field and that $e = 0$ or $e \geqslant n$. Then there is an equivalence of highest weight categories

$$\mathrm{E}_{DJM}^{\Lambda} : \mathcal{S}_n^\Lambda\text{-Mod} \xrightarrow{\sim} \underline{\mathcal{S}}_n^{DJM}\text{-Mod}\,.$$

In particular, up to Morita equivalence, $\underline{\mathcal{S}}_n^{\mathrm{DJM}}$ depends only on e, Λ and the characteristic of $\mathcal{Z}$.

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Suppose that $e = 0$ or $e \geqslant n$. Then S_n^Λ is quasi-hereditary graded cellular algebra.

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Some notations

Recall that

$$\mathcal{A}_0 := \mathbb{Q}(q)[x^{\mathbf{Q}_1}, x^{\mathbf{D}_1}, \cdots, x^{\mathbf{D}_{\ell-1}}]_{(x^{\mathbf{Q}_1}, x^{\mathbf{D}_1}, \cdots, x^{\mathbf{D}_{\ell-1}})},$$
$$\mathcal{O}_0 := \mathbb{Q}[x, q]_{(x)}.$$

We assume that $Q_1 < Q_2 < \cdots < Q_\ell$, and define

$$D_i := Q_{i+1} - Q_1, \ \forall\, 1 \leq i \leq \ell - 1.$$

Let $R \in \{\mathcal{A}_0, \mathcal{O}_0\}$.

Lemma

Let $\lambda, \mu \in \mathcal{P}_n$ and $\mathfrak{s}, \mathfrak{t} \in \mathrm{Std}^{\mu}(\lambda)$. Then the elements in the following set

$$\{\psi^R_{\mathfrak{s},\mathfrak{v}} \mid \mathfrak{s} \in \mathrm{Std}^{\mu}(\lambda), \mathfrak{v} \in \mathrm{Std}(\lambda), \lambda \in \mathcal{P}_n\}$$

form an R-basis of $e^{\mu}_R y^{\mu}_R \mathcal{H}^{\Lambda}_{n,R}$, and the elements in the following set

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A cellular basis for the x-deformation

Definition

Let $\lambda, \mu, \nu \in \mathcal{P}_n$ and $\mathfrak{s} \in \mathrm{Std}^{\mu}(\lambda)$, $\mathfrak{t} \in \mathrm{Std}^{\nu}(\lambda)$. We define

$$\begin{aligned}\Psi^{\mu,\nu}_{\mathfrak{s},\mathfrak{t},R}: \ e^{\nu}_R y^{\nu}_R \mathcal{H}^{\Lambda}_{n,R} &\to e^{\mu}_R y^{\mu}_{\mathcal{O}} \mathcal{H}^{\Lambda}_{n,R} \\ e^{\nu}_R y^{\nu}_R h &\mapsto \psi^{R}_{\mathfrak{s},\mathfrak{t}} h, \quad \forall\, h \in \mathcal{H}^{\Lambda}_{n,R}.\end{aligned}$$

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The elements in $\left\{\Psi^{\mu,\nu}_{\mathfrak{s},\mathfrak{t},R} \mid \mathfrak{s} \in \mathrm{Std}^{\mu}(\lambda), \mathfrak{t} \in \mathrm{Std}^{\nu}(\lambda), \lambda \in \mathcal{P}_n\right\}$ *form an R-basis of* $\mathrm{Hom}_{\mathcal{H}^{\Lambda}_{n,R}}\left(G(\nu)_R, G(\mu)_R\right)$. *Furthermore,*

$$\deg \Psi^{\mu,\nu}_{\mathfrak{s},\mathfrak{t},\mathcal{A}_0} = \deg \mathfrak{s} + \deg \mathfrak{t} - \deg \mathfrak{t}^{\mu} - \deg \mathfrak{t}^{\nu}.$$

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An x-deformation of the cyclotomic quiver Schur algebras

Definition

Let $\beta \in Q_n^+$. We define $S_{\beta,R}^\Lambda := \mathrm{End}_{\mathcal{H}_{n,R}^\Lambda}\left(\bigoplus_{\mu\in\mathcal{P}_\beta} e_R^\mu y_R^\mu \mathcal{H}_{n,R}^\Lambda\right)$.

We shall call $S_{\beta,R}^\Lambda$ an x-deformation of the cyclotomic quiver Schur algebras.

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Let $\beta \in Q_n^+$ and $\mu \in \mathcal{P}_\beta$. Then there exists an element $C_\mu^{\widehat{\mathcal{O}}_0} \in e^\mu y^\mu \mathcal{H}^\Lambda_{\widehat{\mathcal{O}}_0}$ such that the map $e^\mu y^\mu h \mapsto C_\mu^{\widehat{\mathcal{O}}_0} h$ defines a primitive idempotent in $S^\Lambda_{\beta,\widehat{\mathcal{O}}_0}$, and

$$C_\mu^{\widehat{\mathcal{O}}_0} = f^{\mathbb{Q}(x,q)}_{\mathfrak{t}^\mu,\mathfrak{t}^\mu} + \sum_{\substack{\mathfrak{s},\mathfrak{t}\in \mathrm{Std}^\mu(\lambda)\\ \mu \lhd \lambda \in \mathcal{P}_\beta}} a_{\mathfrak{s},\mathfrak{t}} f^{\mathbb{Q}(x,q)}_{\mathfrak{s},\mathfrak{t}},$$

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Applications

In Andrew's talk, for any $\lambda \in \mathcal{P}_n$ and any $\mathfrak{s}, \mathfrak{t} \in \mathrm{Std}(\lambda)$, there is an element $C^{\mathcal{O}_0}_{\mathfrak{s},\mathfrak{t}} \in \mathcal{H}^{\Lambda}_{n,\mathcal{O}_0}$, such that $\{C^{\mathcal{O}_0}_{\mathfrak{s},\mathfrak{t}}\}$ form a cellular $\mathcal{O}_0$-basis of $\mathcal{H}^{\Lambda}_{n,\mathcal{O}_0}$.

Theorem

If $D_1, \cdots, D_{\ell-1}$ are large enough with respect to n, Q_1, then we have that

$$C^{\widehat{\mathcal{O}}_0}_{\mu} \otimes_{\widehat{\mathcal{O}}_0} 1_{\mathbb{Q}(q)} = C^{\mathcal{O}_0}_{\mathfrak{t}^{\mu},\mathfrak{t}^{\mu}} \otimes_{\mathcal{O}_0} 1_{\mathbb{Q}(q)},$$

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and it generates (as a right ideal) the Young module $Y_{\mathbb{Q}(q)}^{\mu}$ of $\mathcal{H}_{n,\mathbb{Q}(q)}^{\Lambda}$.

Applications

In Andrew's talk, for any $\lambda\in\mathcal{P}_n$ and any $\mathfrak{s},\mathfrak{t}\in \mathrm{Std}(\lambda)$, there is an element $C_{\mathfrak{s},\mathfrak{t}}^{\mathcal{O}_0}\in\mathcal{H}_{n,\mathcal{O}_0}^{\Lambda}$, such that $\{C_{\mathfrak{s},\mathfrak{t}}^{\mathcal{O}_0}\}$ form a cellular $\mathcal{O}_0$-basis of $\mathcal{H}_{n,\mathcal{O}_0}^{\Lambda}$.

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Applications (continued)

Let p be a fixed prime number. Recall that

$$\mathcal{O}_p := \mathbb{F}_p[x,q]_{(x)},\ \mathcal{O} := \mathbb{Z}[x,q]_{(x,p)},\ \mathcal{O}_0 := \mathbb{Q}[x,q]_{(x)}.$$

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Note that, in general it is not clear whether $C_{\mathfrak{t}^\mu,\mathfrak{t}^\mu}^{\mathcal{O}_0} \in \mathcal{H}_{n,\mathcal{O}}^{\Lambda}$ or not, not even to say

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Corollary

Let $\lambda \in \mathcal{P}_n$. Then $\dim_q P^\lambda_{\mathbb{Q}(q)} = \dim_q P^\lambda_{\mathbb{F}_p(q)}$ only if $C^{\mathcal{O}_0}_{\mathfrak{t}^\mu,\mathfrak{t}^\mu} \in \mathcal{H}^\Lambda_{n,\mathcal{O}}$ and

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for certain choices of $Q_1, D_1, \cdots, D_{\ell-1}$. In that case,

$$C^{\mathcal{O}_p}_{\mathfrak{t}^\mu,\mathfrak{t}^\mu} = C^{\mathcal{O}_0}_{\mathfrak{t}^\mu,\mathfrak{t}^\mu} \otimes_{\mathcal{O}} 1_{\mathbb{F}_p(q)}.$$

It seems likely that the converse of the above corollary is also true.

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Thank you!