

# List of abstracts

Workshop: Gradings and Decomposition Numbers,  
September 24-28, 2012

All lectures will take place in room V57.02.

## Monday

**13:35-14:25**

### Alexander Kleshchev, Part 1

Title: Representations of KLR algebras of finite and affine type I,II

Abstract: I will survey some results on representations of Khovanov-Lauda-Rouquier algebras focussing on classification of irreducible representations, standard module theory, decomposition numbers, and homological properties. The first talk will be on finite type, and the second talk will be on affine type.

**14:35-15:25**

### Volodymyr Mazorchuk

Title: Endomorphisms of cell 2-representations

Abstract: A fiat 2-category can be considered as a kind of a 2-analogue for a cellular finite dimensional algebra. The classical example of a fiat 2-category is the 2-category of Soergel bimodules over the coinvariant algebra. Cell 2-representations of fiat 2-categories seem to be the most natural candidates for being called “simple” 2-representations. In this talk I plan to explain why, under some natural assumptions, the endomorphism category of such a cell 2-representation is trivial (i.e. equivalent to  $k\text{-mod}$ , where  $k$  is the base field).

If, additionally, the fiat 2-category in question admits a positive grading (like Soergel bimodules do), then the “natural assumptions” can be substantially weakened.

Based on a joint paper with Vanessa Miemietz.

**16:00-16:25**

### Friederike Stoll

Title: Schur-Weyl duality for the Levi subalgebra on mixed tensor space

Abstract: Let  $(n_1, \dots, n_k)$  be a partition of  $n$ . The Levi subalgebra  $L$  of the quantum group  $U_q(\mathfrak{g}_n)$  is the subalgebra corresponding to the Levi subgroup  $GL_{n_1} \times \dots \times GL_{n_k}$  of  $GL_n$ .  $L$  acts naturally on an  $n$ -dimensional space  $V$  and thus on  $V^{\otimes s}$ . We introduce a new algebra generated by certain diagrams and an action thereof on mixed tensor space such that we get a bicentralizer property for these two actions.

**16:35-17:00**

### Michael Ehrig

Title: Diagrammatic description of parabolic category  $\mathcal{O}$  in type  $(D(n), A(n-1))$ .

Abstract: I would like to report on joint work with Catharina Stroppel. We want to introduce a diagram algebra that describes the parabolic category  $\mathcal{O}$  of type  $(D(n), A(n-1))$ . In addition

to a more combinatorial description of the categories, this algebra can be linked to the topology of certain type D Springer fibres as well as various families of algebras, like (cyclotomic) Nazarov-Wenzl algebras and the Brauer algebra.

**17:10-18:00**

**Wolfgang Soergel**

Title: Modular Koszul Duality

Abstract: This is joint work with Simon Riche and Geordie Williamson, in which we extend the Koszul duality from complex category  $\mathcal{O}$  to its positive characteristic analogue. We can do this whenever the characteristic is at least the number of roots plus two, however the statement does not imply the KL-conjectures with any explicit bound.

## Tuesday

**9:00-9:50**

**Catharina Stroppel, Part 1**

Title: Graded Versions of Brauer algebras

Abstract: In this talk I will introduce the (walled) Brauer algebra and study its representation theory depending on the parameter  $\delta$ . I will explain the underlying combinatorics given by certain parabolic Kazhdan-Lusztig polynomials. and finally connect it to cyclotomic quotients of Hecke algebras in a certain limit.

**10:00-10:25**

**Dusko Bogdanic**

Title: Derived equivalences and gradings

Abstract: We show how finite dimensional algebras can be graded via transfer of gradings via derived equivalences. We demonstrate this on the example of Brauer tree algebras and discuss some questions arising from these constructions.

**11:00-11:25**

**Aaron Chan**

Title: Ext-algebra of standard modules for rhombal algebras

Abstract: Rhombal algebra is a family of infinite dimensional quasi-hereditary and symmetric algebras invented by Peach to study blocks of symmetric group (or Schur algebras). I will present some motivations for studying Ext-algebra of standard modules from abstract point of view, and justify the choice of our algebra. We will describe the quiver of the Ext-algebra and even more specifically the (graded) ext-groups and standard modules of rhombal algebras.

**11:35-12:25**

**Leonard L Scott**

Title: Forced gradings,  $p$ -filtrations, and Koszul-like algebras

Abstract: This is joint work with Brian Parshall. A main result is that Weyl modules, for semisimple algebraic groups  $G$  in characteristic  $p > 0$ , have  $p$ -filtrations, when  $p$  is at least  $2h - 2$  (with  $h$  the Coxeter number) and also sufficiently large that the Lusztig character formula holds for all restricted irreducible modules. “ $p$ -filtrations” are defined as having all sections equal to tensor products of restricted irreducible modules with twisted Weyl modules. A main technique involves “forcing” a grading on associated quasi-hereditary algebras by passing to the graded algebras associated to their radical series. Needless to say, a lot of effort is required to make these algebras work in a satisfactory way, even to make them again quasi-hereditary. Perhaps surprisingly, the gradings seem necessary in order to formulate an appropriate  $Ext^1$  criterion for  $p$ -filtrations, and it is even necessary to involve liftings to quantum group module structures. As time permits, I will discuss applications, including instances where the Koszul property can be obtained for such algebras with a forced grading, and, of course—for this conference—some related consequences for decomposition numbers. New objects coming out of these considerations are Koszul-like graded algebras, but with the degree 0 part quasi-hereditary, rather than semisimple.

**15:00-15:50**

**Weiqliang Wang**

Title: Categorification of quantum Superalgebras

Abstract: We categorify one half of a quantum Kac-Moody algebra and superalgebra with non-isotropic odd roots. This inspires a construction of canonical basis for this class of quantum superalgebras for the first time. This is joint work with David Hill and Sean Clark.

**16:30-17:20**

**Andrew Mathas, Part 1**

Title: Cyclotomic quiver Schur algebras and quiver Hecke algebras of type A

Abstract: Building on work of Khovanov, Lauda and Rouquier, Brundan and Kleshchev showed that the cyclotomic Hecke algebras of type A admit a  $\mathbb{Z}$ -grading. They described this grading explicitly by giving a new (homogeneous) presentation for these algebras. The aim of these two talks is to describe how to use this grading to define a graded analogue of the cyclotomic Schur algebras, concentrating on the case of the linear quiver where the Hecke algebra parameter is a non-root of unity. In the first talk I will give the construction of the graded cyclotomic Schur algebras and sketch the proof that these algebras are Koszul in characteristic zero. In the second talk I will explain the close connection between the KLR grading and the more classical seminormal forms. This has several surprising consequences including that the KLR algebra could really have been written down by Young over 100 years ago. The connection with seminormal forms also allows us to construct a remarkable new homogeneous bases of the quiver Hecke algebras and the quiver Schur algebras. This is joint work with Jun Hu and these talks will set the stage for his talk later in the week.

## Wednesday

**9:00-9:50**

**Alexander Kleshchev, Part 2**

Title: Representations of KLR algebras of finite and affine type I,II

Abstract: I will survey some results on representations of Khovanov-Lauda-Rouquier algebras focussing on classification of irreducible representations, standard module theory, decomposition numbers, and homological properties. The first talk will be on finite type, and the second talk will be on affine type.

**10:00-10:25**

**Joseph Loubert**

Title: Affine cellular and affine quasi-hereditary algebras

Abstract: The theories of quasi-hereditary algebras and of cellular algebras have been successful in capturing many of the phenomena encountered in representations of finite dimensional algebras. Recently S. Koenig and C. Xi have defined affine cellular algebras, extending the definition of cellular algebras to nice infinite dimensional algebras. The definition of "affine quasi-hereditary algebra" is still elusive. In this talk I will discuss the affine cellularity of KLR algebras (known in finite type A, and conjectured in all finite types), and why any definition of affine quasi-hereditary should include these algebras.

**11:00-11:25**

**Chris Bowman**

Title: Decomposition numbers of cyclotomic Brauer algebras

Abstract: Given an object of type  $G(m, 1, n)$ , a natural question is whether, in the case that  $m$  is invertible in  $k$ , we can reduce questions about its representation theory to that of a product of objects of type  $G(1, 1, n)$ . We discuss such a reduction theorem in the case of the cyclotomic Brauer algebras of type  $G(m, 1, n)$ . In particular, we obtain the blocks and decomposition numbers of the algebra.

**11:35-12:25**

**David Hemmer**

Title: Generic cohomology and Frobenius twists for symmetric group modules.

Abstract: Let  $U^\lambda$  be a natural module for the symmetric group in characteristic  $p$ , perhaps a Young module, Specht module or irreducible module. We will discuss several recent results of the speaker and others relating  $U^\lambda$  and  $U^{p\lambda}$ . Some of these results are about cohomology of Young and Specht modules, and are reminiscent of generic cohomology for algebraic groups. Others involve  $p$ -Kostka numbers and decompositions of tensor products.

**15:00-15:50**

**Andrew Mathas, Part 2**

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talks is to describe how to use this grading to define a graded analogue of the cyclotomic Schur algebras, concentrating on the case of the linear quiver where the Hecke algebra parameter is a non-root of unity. In the first talk I will give the construction of the graded cyclotomic Schur algebras and sketch the proof that these algebras are Koszul in characteristic zero. In the second talk I will explain the close connection between the KLR grading and the more classical seminormal forms. This has several surprising consequences including that the KLR algebra could really have been written down by Young over 100 years ago. The connection with seminormal forms also allows us to construct a remarkable new homogeneous bases of the quiver Hecke algebras and the quiver Schur algebras. This is joint work with Jun Hu and these talks will set the stage for his talk later in the week.

**16:00-16:25**

**Thangavelu Geetha**

Title: Cellularity of Wreath Product algebras

Abstract: The concept of cellularity of algebras was introduced by Graham and Lehrer in 1996. In this talk we introduce a special class of cellular algebras called cyclic cellular algebras which includes most important classes of cellular algebras that appear in representation theory. For example, Hecke algebras of type  $A$ ,  $q$ -Schur algebras, Brauer algebras and BMW algebras are cyclic cellular. Also we prove that if  $A$  is cyclic cellular algebra, then the wreath product algebras  $A \wr S_n$  are also cyclic cellular. This is a joint work with Frederick M. Goodman.

**17:00-17:50**

**Aaron Lauda, Part 1**

Title: Odd structures arising from categorified quantum groups

Abstract: Khovanov homology is a categorification of the Jones polynomial that paved the way for other categorifications of quantum link invariants. The theory of categorified quantum groups provides a representation theoretic explanation of these homological link invariants via the work of Webster and others. Surprisingly, the categorification of the Jones polynomial is not unique. Ozsvath, Rasmussen, and Szabo introduced an “odd” analog of Khovanov homology that also categorifies the Jones polynomial, and the even and odd categorification are not equivalent. In this talk I will explain joint work with Alexander Ellis and Mikhail Khovanov that aims to develop odd analogs of categorified quantum groups to give a representation theoretic explanation of odd Khovanov homology. These odd categorifications lead to surprising new “odd” structures in geometric representation theory including odd analogs of the cohomology of the Grassmannian and Springer varieties.

## Thursday

**9:00-9:50**

**Gunter Malle**

Title: Local-global conjectures and character bijections

Abstract: We review the recent progress on some of the central conjectures in representation theory of finite groups, in particular their reductions to properties of simple groups. We then

discuss strategies to complete the proofs of these conjectures and draw attention to some open problems in the representation theory of finite groups of Lie type which probably have to be overcome on the way.

**10:00-10:25**

**Qiong Guo**

Title: On the  $U$ -module structure of the unipotent Specht modules for finite general linear groups

Abstract: Let  $q$  be a prime power,  $G = GL_n(q)$  and let  $U \leq G$  be the subgroup of (lower) unitriangular matrices in  $G$ . For a partition  $\lambda$  of  $n$  denote the corresponding unipotent Specht module over the complex field  $\mathbb{C}$  for  $G$  by  $S^\lambda$ . It is conjectured that for  $c \in \mathbb{Z}_{\geq 0}$  the number of irreducible constituents of dimension  $q^c$  of the restriction  $\text{res}_U^G(S^\lambda)$  of  $S^\lambda$  to  $U$  is a polynomial in  $q$  with integer coefficients depending only on  $c$  and  $\lambda$ , not on  $q$ . In the special case of the partition  $\lambda = (1^n)$  this implies a longstanding (still open) conjecture of Higman, stating that the number of conjugacy classes of  $U$  should be a polynomial in  $q$  with integer coefficients depending only on  $n$  not on  $q$ . In this paper we prove the conjecture for the case that  $\lambda = (n - m, m)$  ( $0 \leq m \leq n/2$ ) is a 2-part partition. As a consequence, we obtain a new representation theoretic construction of the standard basis of  $S^\lambda$  (over fields of characteristic coprime to  $q$ ) defined by M. Brandt, R. Dipper, G. James and S. Lyle and an explanation of the rank polynomials appearing there.

**11:00-11:25**

**Sinead Lyle**

Title: Homomorphisms between Specht modules for KLR algebras of type A

Abstract: We discuss some ways of constructing homomorphisms between the Specht modules of the cyclotomic Specht modules for the Khovanov-Lauda-Rouquier algebras of type A. Work of Kelvin Corlett (in the equivalent setting of the Ariki-Koike algebras) gives one approach to this problem. Another approach, based on joint work with Andrew Mathas, will also be discussed.

**11:35-12:25**

**Jun Hu**

Title: Quiver Schur algebras

Abstract: In this talk I shall concentrate on the cyclotomic quiver Hecke and Schur algebras associated to the linear quiver. I will recall the construction of some explicit graded cellular bases of these two algebras, and explain how to define an  $x$ -deformed version of the quiver Schur algebras which turns out to be a (generalised) graded cellular algebra after a suitable completion. This is a joint work with Andrew Mathas.

**15:00-15:50**

**Aaron Lauda, Part 2**

Title: The odd cohomology of Springer varieties and the Hecke algebra at  $q = -1$

The cohomology rings of type A Springer varieties carry an action of the symmetric group (or Hecke algebra at  $q = 1$ ). The top degree cohomology is the Specht module corresponding to the shape of the partition defining the Springer variety. These cohomology rings appear many places in categorified representation theory. In particular, the symmetric group action can be interpreted using the nilHecke algebra which plays a fundamental role in the categorification of  $sl(2)$ . This reinterpretation using the nilHecke algebra allows us to define an odd analog of the cohomology ring of Springer varieties by replacing the nilHecke algebra with the odd nilHecke algebra. The top degree component of the resulting “odd cohomology” of Springer varieties turns out to be isomorphic to a Specht module for the Hecke algebra at  $q = -1$ . (Joint work with Heather Russell)

**16:00-16:25**

**Vladimir Shchigolev**

Title: On decomposition of Bott-Samelson sheaves

Abstract: Recently, Peter Fiebig developed a connection between Lusztig’s conjecture on the characters of irreducible rational representations of reductive algebraic groups over a field  $F$  of positive characteristic and the theory of  $F$ -sheaves on moment graphs. He showed that Lusztig’s conjecture follows from the conjecture on the characters of the Braden–MacPherson sheaves (with coefficients in  $F$ ) on an affine moment graph.

This problem can be reformulated as follows. One takes the trivial sheaf on the affine moment graph and applies to it the translation functors corresponding to a sequence of simple roots  $s$ . The resulting sheaf  $B(s)$  is called a Bott–Samelson sheaf. The main problem is to show that  $B(s)$  decomposes into a sum of Braden–MacPherson (indecomposable) sheaves in the way independent of the characteristic of  $F$  as soon as we have the corresponding GKM-restriction. In my talk, I plan to speak about the exact algorithm to calculate (under some GKM-restriction) the matrix describing the embedding  $B(s)_x \subset B(s)^x$ , where the first module is the costalk and the second one is the stalk at  $x$  of  $B(s)$ . This allows me to calculate the first few terms of the decomposition of  $B(s)$  into a sum of Braden–MacPherson sheaves and to calculate the characters of Braden–MacPherson sheaves in some previously unknown cases.

**17:00-17:50**

**Catharina Stroppel, Part 2**

Title: (Equivariant) cohomology of Springer fibers with applications

Abstract: In this talk I will describe explicitly and in detail the geometry and combinatorics of two-row Springer fibers in type A and D. This leads naturally to a convolution algebra and I will explain the connection to parabolic category  $\mathcal{O}$ . The setup builds on an interesting Langlands duality phenomenon between the Grassmannian of type D and the Springer fiber corresponding to a nilpotent with two Jordan blocks of the same size. If time allows I will state some conjectural connections with representations of  $W$ -algebras.

## Friday

**9:00-9:50**

### **Peter Fiebig**

Title: Critical level representations for affine Kac-Moody algebras.

Abstract: I report on the current status of a joint project with Tomoyuki Arakawa, which aims to prove the Feigin-Frenkel-Lusztig conjecture on the characters of simple highest weight representations of affine Kac-Moody algebras at the critical level. The talk will include an introduction to the conjecture and the relevant categorical framework and, in addition, give an overview on a related structure in positive level. The last part is joint with Martina Lanini.

**10:00-10:25**

### **Johannes Kübel**

Title: Tilting modules and sheaves on moment graphs

Abstract: Due to P. Fiebig, there is an equivalence between a certain subcategory of the deformed BGG category  $\mathcal{O}$  over a semisimple Lie algebra and certain (combinatorial) sheaves on a moment graph, which sends indecomposable projective objects to so-called Braden-MacPherson sheaves corresponding to the Bruhat order on the moment graph. We explain that indecomposable tilting modules of the deformed category  $\mathcal{O}$  map to Braden-MacPherson sheaves constructed along the reversed Bruhat order. If time permits, we will give some applications of this result.

**11:00-11:25**

### **Antonio Sartori**

Title: Categorification of  $gl(1|1)$ -representations.

In this talk I will describe a categorification of tensor powers of the natural representation of the Lie superalgebra  $gl(1|1)$ . This is obtained using certain subcategories of the BGG category  $\mathcal{O}$ , corresponding to properly stratified algebras (a generalization of quasi-hereditary algebras). They encode the combinatorics of canonical bases, that can be computed explicitly through a diagram calculus.

**11:35-12:25**

### **Meinolf Geck**

Title: Cells and decomposition numbers

Abstract: We consider the decomposition matrix of an Iwahori-Hecke algebra associated with a finite Weyl group. The general theory of Kazhdan-Lusztig cells gives rise to a natural ordering of the rows and columns of this matrix. In type  $B_n$  with unequal parameters, this depends on the actual choice of the parameters. In recent joint work with L. Iancu we have given a combinatorial description of these orderings in type  $B_n$ . In the asymptotic case, we recover the dominance order on bipartitions which appeared much earlier in the work of Dipper, James and Murphy.

All lectures will take place in room V57.02.