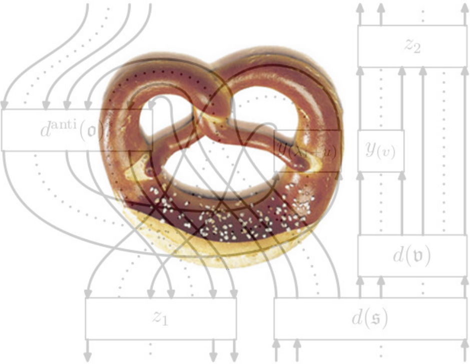
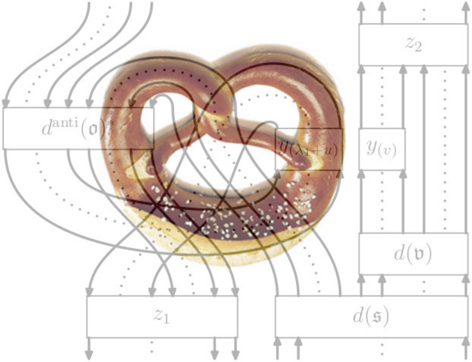
Break!

Break is over!

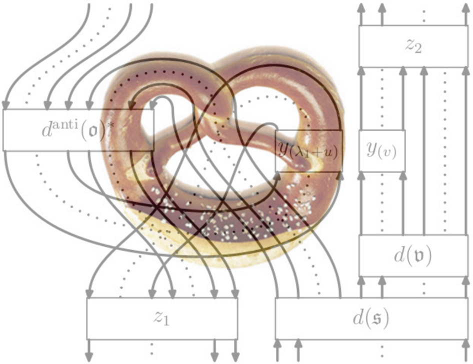
Break is over!



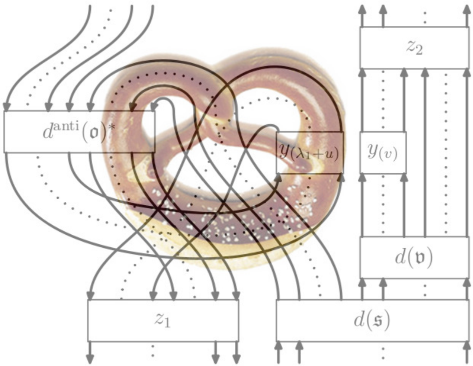
Break is over!



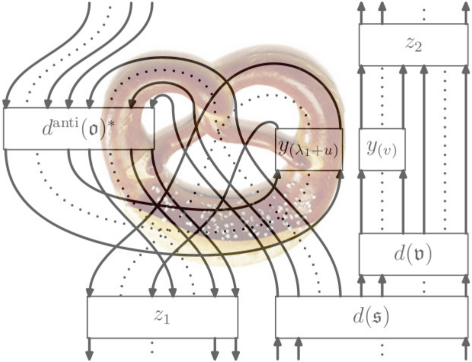
Break is over!



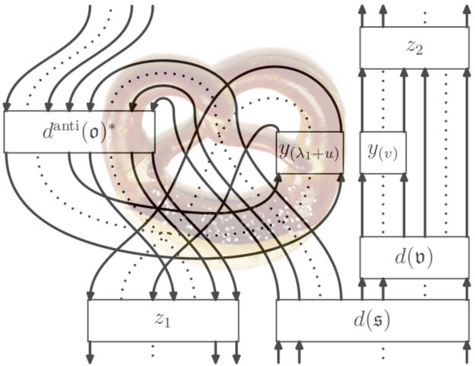
Break is over!



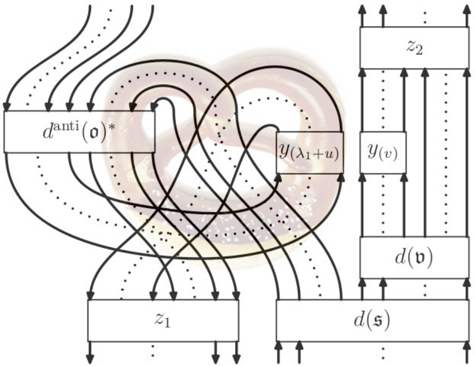
Break is over!



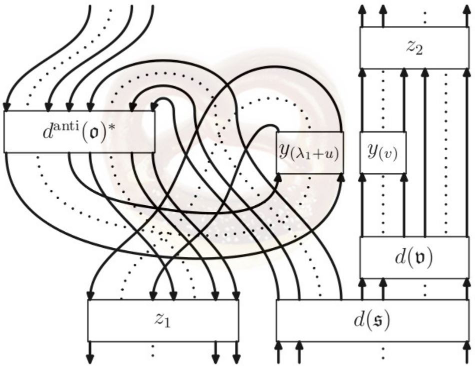
Break is over!



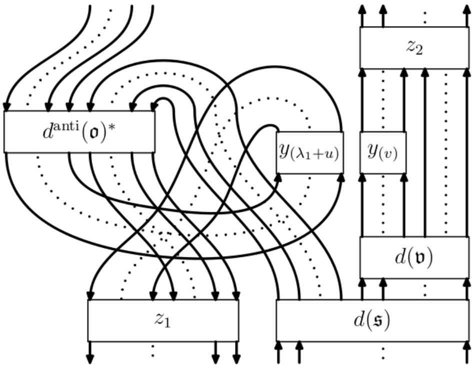
Break is over!



Break is over!



Break is over!



Break is over!

A cell filtration of mixed tensor space, part II

Mathias Werth
joint work with Friederike Stoll

Institut für Algebra und Zahlentheorie
Universität Stuttgart

Stuttgart, 12 September 2014

the Littlewood-Richardson rule

- Let $V(\lambda, \mu)$ be the irreducible rational $U(\mathfrak{gl}_n)$-module attached to the pair of partitions (λ, μ).
- $V = V(\Box, -)$ and $V^* = V(-, \Box)$.

the Littlewood-Richardson rule

- Let $V(\lambda, \mu)$ be the irreducible rational $U(\mathfrak{gl}_n)$-module attached to the pair of partitions (λ, μ).
- $V = V(\square, -)$ and $V^* = V(-, \square)$.
- $V(\lambda, \mu) \otimes V = \bigoplus_{(\lambda', \mu')} V(\lambda', \mu')$

the Littlewood-Richardson rule

- Let $V(\lambda, \mu)$ be the irreducible rational $U(\mathfrak{gl}_n)$-module attached to the pair of partitions (λ, μ).
- $V = V(\square, -)$ and $V^* = V(-, \square)$.
- $V(\lambda, \mu) \otimes V = \bigoplus_{(\lambda', \mu')} V(\lambda', \mu')$ where the sum is taken over all (λ', μ') with either λ' is λ plus one box or μ' is μ minus one box,

the Littlewood-Richardson rule

- Let $V(\lambda, \mu)$ be the irreducible rational $U(\mathfrak{gl}_n)$-module attached to the pair of partitions (λ, μ).
- $V = V(\Box, -)$ and $V^* = V(-, \Box)$.
- $V(\lambda, \mu) \otimes V = \bigoplus_{(\lambda', \mu')} V(\lambda', \mu')$ where the sum is taken over all (λ', μ') with either λ' is λ plus one box or μ' is μ minus one box, such that there are not more than n boxes in the first row.
- $V(\lambda, \mu) \otimes V^* = \bigoplus_{(\lambda', \mu')} V(\lambda', \mu')$

the Littlewood-Richardson rule

- Let $V(\lambda,\mu)$ be the irreducible rational $U(\mathfrak{gl}_n)$-module attached to the pair of partitions (λ,μ).
- $V = V(\square, -)$ and $V^* = V(-,\square)$.
- $V(\lambda,\mu)\otimes V = \bigoplus_{(\lambda',\mu')} V(\lambda',\mu')$ where the sum is taken over all (λ',μ') with either λ' is λ plus one box or μ' is μ minus one box, such that there are not more than n boxes in the first row.
- $V(\lambda,\mu)\otimes V^* = \bigoplus_{(\lambda',\mu')} V(\lambda',\mu')$ where the sum is taken over all (λ',μ') with either λ' is λ minus one box or μ' is μ plus one box, such that there are not more than n boxes in the first row.

Example ($n \geq 4$)

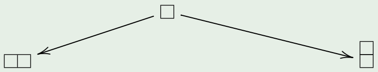
V $\square$

the Littlewood-Richardson rule

Example ($n \geq 4$)

V

$V^{\otimes 2}$



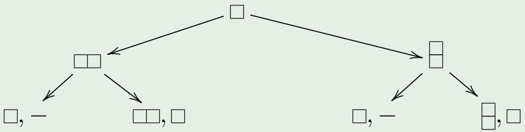
the Littlewood-Richardson rule

Example ($n \geq 4$)

V

$V^{\otimes 2}$

$V^{\otimes 2} \otimes V^*$



the Littlewood-Richardson rule

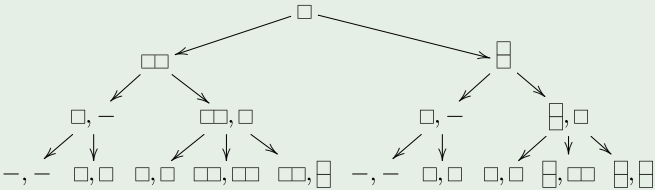
Example ($n \geq 4$)

V

$V^{\otimes 2}$

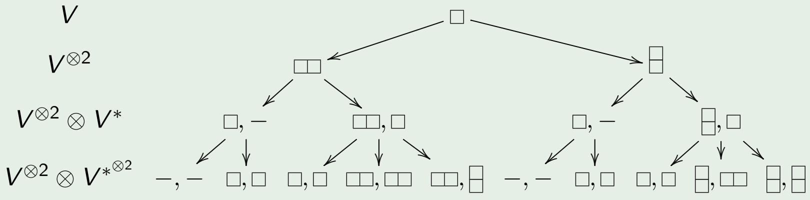
$V^{\otimes 2} \otimes V^*$

$V^{\otimes 2} \otimes {V^*}^{\otimes 2}$



the Littlewood-Richardson rule

Example ($n \geq 4$)



The multiplicity of the $U(\mathfrak{gl}_4)$-modules $V(\lambda, \mu)$ in $V^{\otimes 2} \otimes V^{*\otimes 2}$ equals the number of paths to (λ, μ).

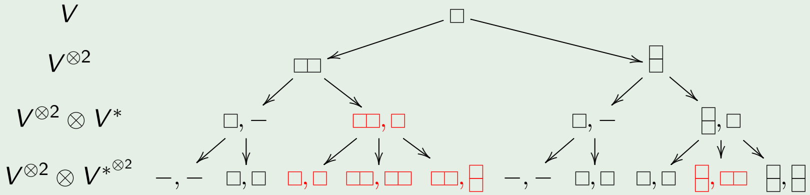
$\rightsquigarrow$ a basis of $B_{2,2}$ can be indexed by tuples of paths (λ, μ).

Example ($n = 2$)

Cancel all paths involving pairs (λ, μ) with $\lambda_1 + \mu_1 > 2$

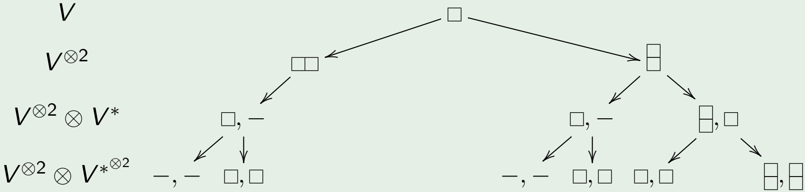
Example ($n = 2$)

Cancel all paths involving pairs (λ, μ) with $\lambda_1 + \mu_1 > 2$



Example ($n = 2$)

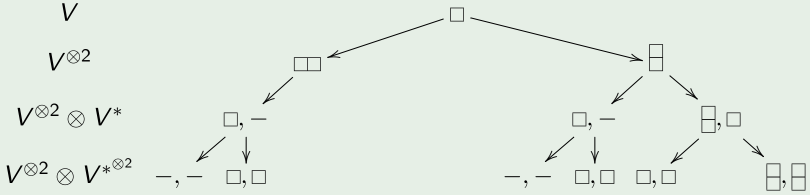
Cancel all paths involving pairs (λ, μ) with $\lambda_1 + \mu_1 > 2$



A basis of $B_{2,2}(n)$/annihilator can be indexed by tuples of these paths.

Example ($n = 2$)

Cancel all paths involving pairs (λ, μ) with $\lambda_1 + \mu_1 > 2$



A basis of $B_{2,2}(n)$/annihilator can be indexed by tuples of these paths.
$\rightsquigarrow$ it SHOULD be indexed by tuples of paths!

tableaux?

Example

A path to $\left(\yng(2,2,1),\yng(3,1,1)\right)$ of length $9+9$

Example

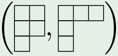
A path to $\left(\ydiagram{2,2,1}, \ydiagram{3,1,1}\right)$ of length $9 + 9$

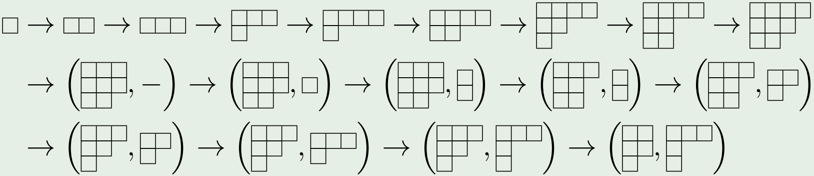
$$\ydiagram{1} \to \ydiagram{2} \to \ydiagram{3} \to \ydiagram{3,1} \to \ydiagram{4,1} \to \ydiagram{4,2} \to \ydiagram{4,2,1} \to \ydiagram{4,2,2} \to \ydiagram{4,3,2}$$

$$\to \left(\ydiagram{3,3,2}, -\right) \to \left(\ydiagram{3,3,2}, \ydiagram{1}\right) \to \left(\ydiagram{3,3,2}, \ydiagram{1,1}\right) \to \left(\ydiagram{3,2,2}, \ydiagram{1,1}\right) \to \left(\ydiagram{3,2,2}, \ydiagram{2,1}\right)$$

$$\to \left(\ydiagram{3,2,1}, \ydiagram{2,1}\right) \to \left(\ydiagram{3,2,1}, \ydiagram{3,1}\right) \to \left(\ydiagram{3,2,1}, \ydiagram{3,1,1}\right) \to \left(\ydiagram{2,2,1}, \ydiagram{3,1,1}\right)$$

Example

A path to  of length $9+9$



We attach to it the following triple of tableaux

Example

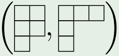
A path to $\left(\yng(2,2,1), \yng(3,1,1)\right)$ of length $9+9$

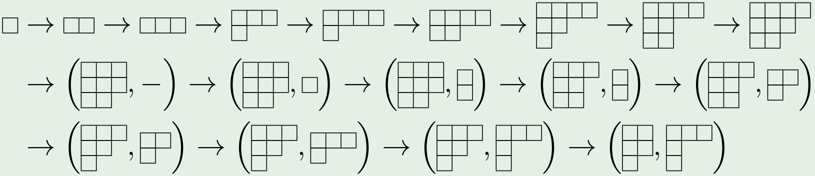
$$\yng(1) \to \yng(2) \to \yng(3) \to \yng(3,1) \to \yng(4,1) \to \yng(4,2) \to \yng(4,2,1) \to \yng(4,2,2) \to \yng(4,3,2)$$
$$\to \left(\yng(3,3,2), -\right) \to \left(\yng(3,3,2), \yng(1)\right) \to \left(\yng(3,3,2), \yng(1,1)\right) \to \left(\yng(3,2,2), \yng(1,1)\right) \to \left(\yng(3,2,2), \yng(2,1)\right)$$
$$\to \left(\yng(3,2,1), \yng(2,1)\right) \to \left(\yng(3,2,1), \yng(3,1)\right) \to \left(\yng(3,2,1), \yng(3,1,1)\right) \to \left(\yng(2,2,1), \yng(3,1,1)\right)$$

We attach to it the following triple of tableaux

$$\left(\young(1235,469,78)\,,\right.$$

Example

A path to  of length $9+9$



We attach to it the following triple of tableaux

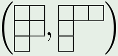
1	2	3	5
4	6	9	
7	8		

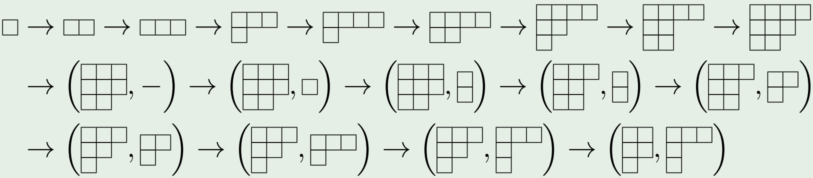
,

		9	1
		4	
	6		

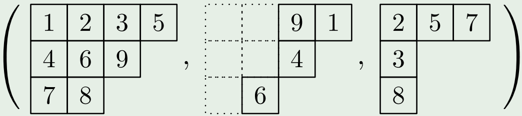
,

Example

A path to  of length $9 + 9$



We attach to it the following triple of tableaux



triples of tableaux!

Standard triples

$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v})$ is a standard triple of shape (λ, μ) if

triples of tableaux!

Standard triples

$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v})$ is a standard triple of shape (λ, μ) if

- $\mathfrak{t}$ is a standard ν-tableau with entries $\{1, \ldots, r\}$, ν a partition of r,

triples of tableaux!

Standard triples

$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v})$ is a standard triple of shape (λ, μ) if

- $\mathfrak{t}$ is a standard ν-tableau with entries $\{1, \ldots, r\}$, ν a partition of r,
- $\mathfrak{u}$ is an anti-standard ρ-tableau, s.t. $[\rho] \subseteq [\nu]$ and $[\nu] \setminus [\rho] = [\lambda]$,

Standard triples

$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v})$ is a standard triple of shape (λ, μ) if

- $\mathfrak{t}$ is a standard ν-tableau with entries $\{1, \ldots, r\}$, ν a partition of r,
- $\mathfrak{u}$ is an anti-standard ρ-tableau, s.t. $[\rho] \subseteq [\nu]$ and $[\nu] \setminus [\rho] = [\lambda]$,
- $\mathfrak{v}$ is a standard μ-tableau,

triples of tableaux!

Standard triples

$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v})$ is a standard triple of shape (λ, μ) if

- $\mathfrak{t}$ is a standard ν-tableau with entries $\{1, \ldots, r\}$, ν a partition of r,
- $\mathfrak{u}$ is an anti-standard ρ-tableau, s.t. $[\rho] \subseteq [\nu]$ and $[\nu] \setminus [\rho] = [\lambda]$,
- $\mathfrak{v}$ is a standard μ-tableau,
- the entries of $\mathfrak{u}$ and $\mathfrak{v}$ are $\{1, \ldots, s\}$.

Example

Consider the standard triple

$$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v}) = \left(\begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 & 9 \\ \cline{1-3} 7 & 8 \\ \cline{1-2} \end{array} \, , \; \begin{array}{cccc} \cdot & \cdot & \boxed{9} & \boxed{1} \\ \cdot & \cdot & \boxed{4} & \\ \cdot & \boxed{6} & & \end{array} \, , \; \begin{array}{|c|c|c|} \hline 2 & 5 & 7 \\ \hline 3 \\ \cline{1-1} 8 \\ \cline{1-1} \end{array} \right).$$

Example

Consider the standard triple

$$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v}) = \left(\begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 & 9 \\ \cline{1-3} 7 & 8 \\ \cline{1-2} \end{array} , \begin{array}{cccc} \cdot & \cdot & \boxed{9} & \boxed{1} \\ \cdot & \cdot & \boxed{4} & \\ \cdot & \boxed{6} & & \end{array} , \begin{array}{|c|c|c|} \hline 2 & 5 & 7 \\ \hline 3 \\ \cline{1-1} 8 \\ \cline{1-1} \end{array} \right).$$

We need two auxiliary tableaux to define the basis elements:

Example

Consider the standard triple

$$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v}) = \left(\begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 & 9 \\ \cline{1-3} 7 & 8 \\ \cline{1-2} \end{array}, \begin{array}{cccc} & & \multicolumn{1}{|c|}{9} & \multicolumn{1}{c|}{1} \\ \cline{3-4} & & \multicolumn{1}{|c|}{4} \\ \cline{2-3} & \multicolumn{1}{|c|}{6} \\ \cline{2-2} \end{array}, \begin{array}{|c|c|c|} \hline 2 & 5 & 7 \\ \hline 3 \\ \cline{1-1} 8 \\ \cline{1-1} \end{array} \right).$$

We need two auxiliary tableaux to define the basis elements:

$$\mathfrak{o} = \begin{array}{|c|c|c|c|} \hline 14 & 13 & 9 & 1 \\ \hline 12 & 11 & 4 \\ \cline{1-3} 10 & 6 \\ \cline{1-2} \end{array}$$

almost quintuples

Example

Consider the standard triple

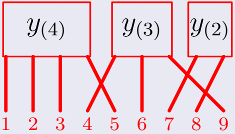
$$(\mathfrak{t}, \mathfrak{u}, \mathfrak{v}) = \left(\begin{array}{|c|c|c|c|} \hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 & 9 \\ \cline{1-3} 7 & 8 \\ \cline{1-2} \end{array} \, , \, \begin{array}{cccc} & & 9 & 1 \\ & & 4 & \\ & 6 & & \end{array} \, , \, \begin{array}{|c|c|c|} \hline 2 & 5 & 7 \\ \hline 3 \\ \cline{1-1} 8 \\ \cline{1-1} \end{array} \right).$$

We need two auxiliary tableaux to define the basis elements:

$$\mathfrak{o} = \begin{array}{|c|c|c|c|} \hline 14 & 13 & 9 & 1 \\ \hline 12 & 11 & 4 \\ \cline{1-3} 10 & 6 \\ \cline{1-2} \end{array} \quad \text{and} \quad \mathfrak{s} = \begin{array}{|c|c|c|c|c|} \cline{1-4} 1 & 4 & 6 & 9 \\ \hline 2 & 3 & 5 & 7 & 8 \\ \hline \end{array}$$

basis elements, first half

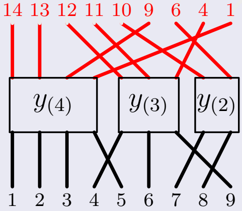
$$\mathfrak{t} = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 & 9 \\ \cline{1-3} 7 & 8 \\ \cline{1-2}\end{array}$$



basis elements, first half

$$\mathfrak{t} = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 & 9 & \\ \cline{1-3} 7 & 8 & & \\ \cline{1-2} \end{array}$$

$$\mathfrak{s} = \begin{array}{|c|c|c|c|}\hline 14 & 13 & 9 & 1 \\ \hline 12 & 11 & 4 & \\ \cline{1-3} 10 & 6 & & \\ \cline{1-2} \end{array}$$



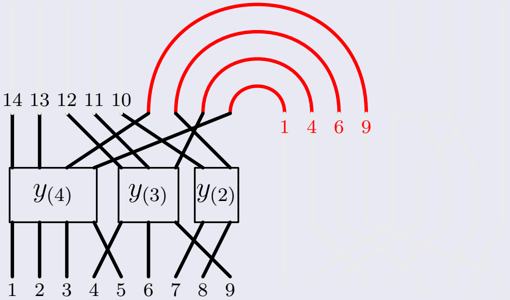
basis elements, first half

$\mathfrak{t} =$

1	2	3	5
4	6	9	
7	8		

$\mathfrak{o} =$

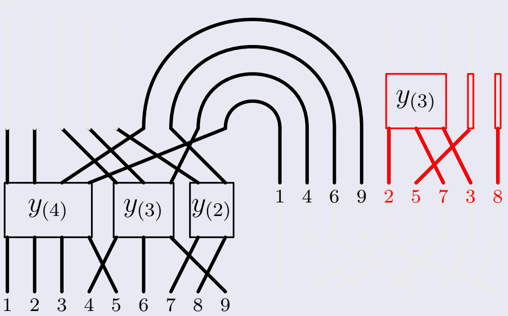
14	13	9	1
12	11	4	
10	6		



$$\mathfrak{t} = \begin{array}{|c|c|c|c|}\hline 1 & 2 & 3 & 5 \\ \hline 4 & 6 & 9 \\ \cline{1-3} 7 & 8 \\ \cline{1-2}\end{array}$$

$$\mathfrak{o} = \begin{array}{|c|c|c|c|}\hline 14 & 13 & 9 & 1 \\ \hline 12 & 11 & 4 \\ \cline{1-3} 10 & 6 \\ \cline{1-2}\end{array}$$

$$\mathfrak{v} = \begin{array}{|c|c|c|}\hline 2 & 5 & 7 \\ \hline 3 \\ \cline{1-1} 8 \\ \cline{1-1}\end{array}$$



basis elements, first half

$\mathfrak{t} =$

1	2	3	5
4	6	9	
7	8		

$\mathfrak{o} =$

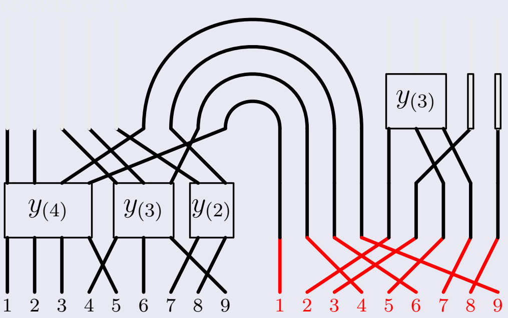
14	13	9	1
12	11	4	
10	6		

$\mathfrak{v} =$

2	5	7
3		
8		

$\mathfrak{s} =$

1	4	6	9	
2	3	5	7	8



basis elements, first half

$\mathfrak{t} =$

1	2	3	5
4	6	9	
7	8		

$\mathfrak{o} =$

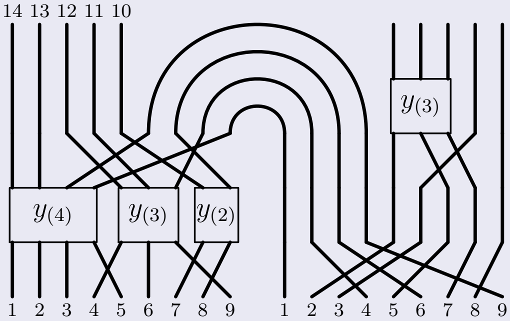
14	13	9	1
12	11	4	
10	6		

$\mathfrak{v} =$

2	5	7
3		
8		

$\mathfrak{s} =$

1	4	6	9	
2	3	5	7	8



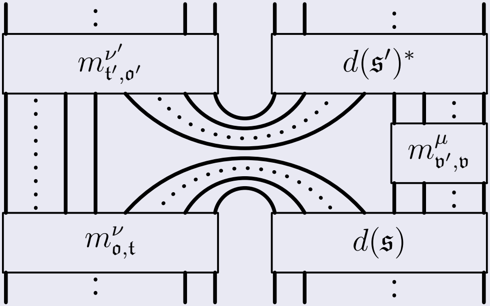
basis elements

With this we are able to define elements in the walled Brauer algebra

basis elements

With this we are able to define elements in the walled Brauer algebra

$m_{(\mathfrak{t}',\mathfrak{u}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v})} := c$



Theorem (Stoll, W)

The set

$$\left\{ m_{(\mathfrak{t}',\mathfrak{s}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v})} \;\middle|\; \begin{array}{l} (\mathfrak{t}',\mathfrak{s}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v}) \textit{ standard triples of} \\ \textit{shape } (\lambda,\mu),(\lambda,\mu)\in\Lambda(r,s) \end{array} \right\}$$

Theorem (Stoll, W)

The set

$$\left\{ m_{(\mathfrak{t}',\mathfrak{s}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v})} \;\middle|\; \begin{array}{l} (\mathfrak{t}',\mathfrak{s}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v}) \textit{ standard triples of} \\ \textit{shape } (\lambda,\mu),(\lambda,\mu) \in \Lambda(r,s) \end{array} \right\}$$

is a cellular basis of the walled Brauer algebra $B_{r,s}(x)$. The partial order on $\Lambda(r,s)$ is given by $\trianglerighteq$.

a new cellular basis

Theorem (Stoll, W)

The set

$$\left\{ m_{(\mathfrak{t}',\mathfrak{s}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v})} \;\middle|\; \begin{array}{l} (\mathfrak{t}',\mathfrak{s}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v}) \textit{ standard triples of} \\ \textit{shape } (\lambda,\mu),(\lambda,\mu) \in \Lambda(r,s) \end{array} \right\}$$

is a cellular basis of the walled Brauer algebra $B_{r,s}(x)$. The partial order on $\Lambda(r,s)$ is given by $\trianglerighteq$.

The set

$$\left\{ m_{(\mathfrak{t},\mathfrak{u},\mathfrak{v})} \;\middle|\; (\mathfrak{t},\mathfrak{u},\mathfrak{v}) \text{ standard triples of shape } (\lambda,\mu) \right\}$$

Theorem (Stoll, W)

The set

$$\left\{ m_{(\mathfrak{t}',\mathfrak{s}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v})} \;\middle|\; \begin{array}{l} (\mathfrak{t}',\mathfrak{s}',\mathfrak{v}'),(\mathfrak{t},\mathfrak{u},\mathfrak{v}) \textit{ standard triples of} \\ \textit{shape } (\lambda,\mu),(\lambda,\mu)\in\Lambda(r,s) \end{array} \right\}$$

is a cellular basis of the walled Brauer algebra $B_{r,s}(x)$. The partial order on $\Lambda(r,s)$ is given by $\trianglerighteq$.

The set

$$\left\{ m_{(\mathfrak{t},\mathfrak{u},\mathfrak{v})} \;\middle|\; (\mathfrak{t},\mathfrak{u},\mathfrak{v}) \text{ standard triples of shape } (\lambda,\mu) \right\}$$

is a basis for the cell module $C(\lambda,\mu)$ of $B_{r,s}(x)$.

Restriction of cell modules

- $C(\lambda, \mu)$ a cell module of $B_{r,s}(x)$,

Restriction of cell modules

- $C(\lambda, \mu)$ a cell module of $B_{r,s}(x)$,
  Res $C(\lambda, \mu)$ the restriction of $C(\lambda, \mu)$ to $B_{r,s-1}(x)$

properties of the new basis

Restriction of cell modules

- $C(\lambda, \mu)$ a cell module of $B_{r,s}(x)$,
  Res $C(\lambda, \mu)$ the restriction of $C(\lambda, \mu)$ to $B_{r,s-1}(x)$
- Res $C(\lambda, \mu)$ possesses a filtration of cell modules for $B_{r,s-1}(x)$, the new basis is adapted to this filtration.

properties of the new basis

Restriction of cell modules

- $C(\lambda, \mu)$ a cell module of $B_{r,s}(x)$,
  Res $C(\lambda, \mu)$ the restriction of $C(\lambda, \mu)$ to $B_{r,s-1}(x)$
- Res $C(\lambda, \mu)$ possesses a filtration of cell modules for $B_{r,s-1}(x)$, the new basis is adapted to this filtration.
- the isomorphisms between factors and cell modules for the smaller algebra can be described easily using the cell bases.

Example (Res $C(\square, \square\square)$)

the Restriction of cell modules

Example (Res $C(\square, \square\square)$)

$$U(\trianglerighteq (\square, \square)) \left\{ \begin{array}{l} m_{\left(\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array}, \begin{array}{c} \cdot \\ \hline \multicolumn{1}{|c|}{2} \\ \hline \end{array}, \begin{array}{|c|c|}\hline 1 & 3 \\ \hline \end{array}\right)} \\ m_{\left(\begin{array}{|c|}\hline 1 \\ \hline 2 \\ \hline \end{array}, \begin{array}{c} \cdot \\ \hline \multicolumn{1}{|c|}{1} \\ \hline \end{array}, \begin{array}{|c|c|}\hline 2 & 3 \\ \hline \end{array}\right)} \\ m_{\left(\begin{array}{|c|c|}\hline 1 & 2 \\ \hline \end{array}, \begin{array}{cc} \cdot & \multicolumn{1}{|c|}{2} \\ \end{array}, \begin{array}{|c|c|}\hline 1 & 3 \\ \hline \end{array}\right)} \\ m_{\left(\begin{array}{|c|c|}\hline 1 & 2 \\ \hline \end{array}, \begin{array}{cc} \cdot & \multicolumn{1}{|c|}{1} \\ \end{array}, \begin{array}{|c|c|}\hline 2 & 3 \\ \hline \end{array}\right)} \end{array} \right.$$

Example (Res $C(\square, \square\square)$)

$$
U(\trianglerighteq (\square,\square)) \left\{
\begin{array}{lcl}
m_{\left(\begin{smallmatrix}\boxed{1}\\ \boxed{2}\end{smallmatrix},\ \begin{smallmatrix}\cdot\\ \boxed{2}\end{smallmatrix},\ \boxed{1}\boxed{3}\right)} & \mapsto & m_{\left(\begin{smallmatrix}\boxed{1}\\ \boxed{2}\end{smallmatrix},\ \begin{smallmatrix}\cdot\\ \boxed{2}\end{smallmatrix},\ \boxed{1}\right)} \\
m_{\left(\begin{smallmatrix}\boxed{1}\\ \boxed{2}\end{smallmatrix},\ \begin{smallmatrix}\cdot\\ \boxed{1}\end{smallmatrix},\ \boxed{2}\boxed{3}\right)} & \mapsto & m_{\left(\begin{smallmatrix}\boxed{1}\\ \boxed{2}\end{smallmatrix},\ \begin{smallmatrix}\cdot\\ \boxed{1}\end{smallmatrix},\ \boxed{2}\right)} \\
m_{\left(\boxed{1}\boxed{2},\ \cdot\,\boxed{2},\ \boxed{1}\boxed{3}\right)} & \mapsto & m_{\left(\boxed{1}\boxed{2},\ \cdot\,\boxed{2},\ \boxed{1}\right)} \\
m_{\left(\boxed{1}\boxed{2},\ \cdot\,\boxed{1},\ \boxed{2}\boxed{3}\right)} & \mapsto & m_{\left(\boxed{1}\boxed{2},\ \cdot\,\boxed{1},\ \boxed{2}\right)}
\end{array}
\right\} C(\square,\square)
$$

Example (Res $C(\square, \square\square)$)

$$U(\trianglerighteq (\square\square, \square\square)) \left\{ \begin{array}{lcl} m_{\left(\ytableaushort{12}, \ytableaushort{\none 3}, \ytableaushort{12}\right)} + \dots & & \\ m_{\left(\ytableaushort{1,2}, \ytableaushort{\none,2}, \ytableaushort{13}\right)} & \mapsto & m_{\left(\ytableaushort{1,2}, \ytableaushort{\none,2}, \ytableaushort{1}\right)} \\ m_{\left(\ytableaushort{1,2}, \ytableaushort{\none,1}, \ytableaushort{23}\right)} & \mapsto & m_{\left(\ytableaushort{1,2}, \ytableaushort{\none,1}, \ytableaushort{2}\right)} \\ m_{\left(\ytableaushort{12}, \ytableaushort{\none 2}, \ytableaushort{13}\right)} & \mapsto & m_{\left(\ytableaushort{12}, \ytableaushort{\none 2}, \ytableaushort{1}\right)} \\ m_{\left(\ytableaushort{12}, \ytableaushort{\none 1}, \ytableaushort{23}\right)} & \mapsto & m_{\left(\ytableaushort{12}, \ytableaushort{\none 1}, \ytableaushort{2}\right)} \end{array} \right. \left. \vphantom{\begin{array}{l} m \\ m \\ m \\ m \end{array}} \right\} C(\square, \square)$$

Example (Res $C(\square, \square\square)$)

$$U(\trianglerighteq (\square\square, \square\square)) \begin{cases} m_{\left(\boxed{1}\boxed{2},\ \cdot\boxed{3},\ \boxed{1}\boxed{2}\right)} + \ldots & \mapsto \quad m_{\left(\boxed{1}\boxed{2},\ -,\ \boxed{1}\boxed{2}\right)} \\ m_{\left(\begin{smallmatrix}\boxed{1}\\ \boxed{2}\end{smallmatrix},\ \begin{smallmatrix}\cdot\\ \boxed{2}\end{smallmatrix},\ \boxed{1}\boxed{3}\right)} & \mapsto \quad m_{\left(\begin{smallmatrix}\boxed{1}\\ \boxed{2}\end{smallmatrix},\ \begin{smallmatrix}\cdot\\ \boxed{2}\end{smallmatrix},\ \boxed{1}\right)} \\ m_{\left(\begin{smallmatrix}\boxed{1}\\ \boxed{2}\end{smallmatrix},\ \begin{smallmatrix}\cdot\\ \boxed{1}\end{smallmatrix},\ \boxed{2}\boxed{3}\right)} & \mapsto \quad m_{\left(\begin{smallmatrix}\boxed{1}\\ \boxed{2}\end{smallmatrix},\ \begin{smallmatrix}\cdot\\ \boxed{1}\end{smallmatrix},\ \boxed{2}\right)} \\ m_{\left(\boxed{1}\boxed{2},\ \cdot\boxed{2},\ \boxed{1}\boxed{3}\right)} & \mapsto \quad m_{\left(\boxed{1}\boxed{2},\ \cdot\boxed{2},\ \boxed{1}\right)} \\ m_{\left(\boxed{1}\boxed{2},\ \cdot\boxed{1},\ \boxed{2}\boxed{3}\right)} & \mapsto \quad m_{\left(\boxed{1}\boxed{2},\ \cdot\boxed{1},\ \boxed{2}\right)} \end{cases}$$

where the first image row spans $C(\square\square, \square\square)$ and the remaining four image rows span $C(\square, \square)$.

Example (Res $C(\square, \square\square)$)

$$
U\left(\trianglerighteq \left(\ydiagram{1,1}, \ydiagram{2}\right)\right)
\begin{cases}
m_{\left(\ytableaushort{1,2}, \ytableaushort{\none[\cdot],3}, \ytableaushort{12}\right)} + \dots & \\
m_{\left(\ytableaushort{12}, \ytableaushort{\none[\cdot]3}, \ytableaushort{12}\right)} + \dots & \mapsto \quad m_{\left(\ytableaushort{12}, -, \ytableaushort{12}\right)} \quad \Big\} \; C(\square\square, \square\square) \\
m_{\left(\ytableaushort{1,2}, \ytableaushort{\none[\cdot],2}, \ytableaushort{13}\right)} & \mapsto \quad m_{\left(\ytableaushort{1,2}, \ytableaushort{\none[\cdot],2}, \ytableaushort{1}\right)} \\
m_{\left(\ytableaushort{1,2}, \ytableaushort{\none[\cdot],1}, \ytableaushort{23}\right)} & \mapsto \quad m_{\left(\ytableaushort{1,2}, \ytableaushort{\none[\cdot],1}, \ytableaushort{2}\right)} \\
m_{\left(\ytableaushort{12}, \ytableaushort{\none[\cdot]2}, \ytableaushort{13}\right)} & \mapsto \quad m_{\left(\ytableaushort{12}, \ytableaushort{\none[\cdot]2}, \ytableaushort{1}\right)} \\
m_{\left(\ytableaushort{12}, \ytableaushort{\none[\cdot]1}, \ytableaushort{23}\right)} & \mapsto \quad m_{\left(\ytableaushort{12}, \ytableaushort{\none[\cdot]1}, \ytableaushort{2}\right)}
\end{cases}
\left.\vphantom{\begin{matrix}a\\a\\a\\a\end{matrix}}\right\} C(\square, \square)
$$

(The last brace groups the final four mapped terms under $C(\square, \square)$.)

the Restriction of cell modules

Example (Res $C(\ydiagram{1}, \ydiagram{2})$)

$$
U\left(\trianglerighteq \left(\ydiagram{1,1}, \ydiagram{2}\right)\right)
\begin{cases}
m_{\left(\ytableaushort{1,2},\, \ytableaushort{\none,3},\, \ytableaushort{12}\right)} + \dots & \mapsto \quad m_{\left(\ytableaushort{1,2},\, -,\, \ytableaushort{12}\right)} \\
m_{\left(\ytableaushort{12},\, \ytableaushort{\none 3},\, \ytableaushort{12}\right)} + \dots & \mapsto \quad m_{\left(\ytableaushort{12},\, -,\, \ytableaushort{12}\right)} \\
m_{\left(\ytableaushort{1,2},\, \ytableaushort{\none,2},\, \ytableaushort{13}\right)} & \mapsto \quad m_{\left(\ytableaushort{1,2},\, \ytableaushort{\none,2},\, \ytableaushort{1}\right)} \\
m_{\left(\ytableaushort{1,2},\, \ytableaushort{\none,1},\, \ytableaushort{23}\right)} & \mapsto \quad m_{\left(\ytableaushort{1,2},\, \ytableaushort{\none,1},\, \ytableaushort{2}\right)} \\
m_{\left(\ytableaushort{12},\, \ytableaushort{\none 2},\, \ytableaushort{13}\right)} & \mapsto \quad m_{\left(\ytableaushort{12},\, \ytableaushort{\none 2},\, \ytableaushort{1}\right)} \\
m_{\left(\ytableaushort{12},\, \ytableaushort{\none 1},\, \ytableaushort{23}\right)} & \mapsto \quad m_{\left(\ytableaushort{12},\, \ytableaushort{\none 1},\, \ytableaushort{2}\right)}
\end{cases}
$$

- Row 1: $\}\; C(\ydiagram{1,1}, \ydiagram{2})$
- Row 2: $\}\; C(\ydiagram{2}, \ydiagram{2})$
- Rows 3–6: $\}\; C(\ydiagram{1}, \ydiagram{1})$

Theorem (DIPPER, DOTY, STOLL)

There is an isomorphism of vector spaces

$$\mathbb{F}\mathfrak{S}_{r+s} \to B_{r,s}(n),$$

the annihilator

Theorem (Dipper, Doty, Stoll)

There is an isomorphism of vector spaces

$$\mathbb{F}\mathfrak{S}_{r+s} \to B_{r,s}(n),$$

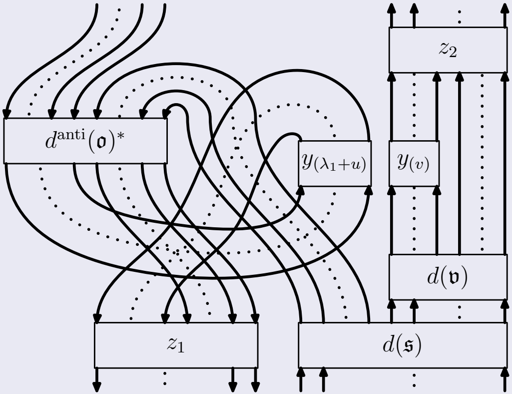
such that the annihilator of the tensor space is mapped to the annihilator of the mixed tensor space ($n = \dim V$).

Our basis is not yet adjusted to annihilators. In order to achieve this we attach a number to each standard triple.

Our basis is not yet adjusted to annihilators. In order to achieve this we attach a number to each standard triple.
Let $\max(\mathfrak{t}, \mathfrak{u}, \mathfrak{v})$ be the maximal amount of boxes in the first row of the pairs of diagrams in the corresponding path.

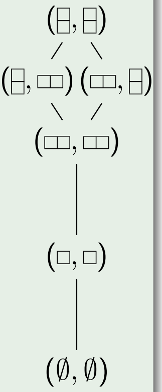
With this we are able to further adjust our basis:

With this we are able to further adjust our basis:



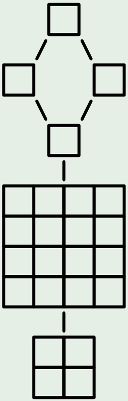
the annihilator

Example (The annihilator in $B_{2,2}(n)$, $n = \dim(V)$)

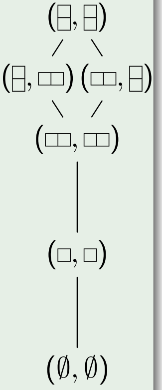


the annihilator

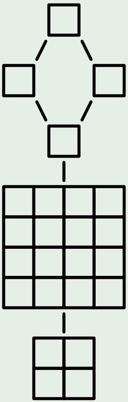
Example (The annihilator in $B_{2,2}(n)$, $n = \dim(V)$)



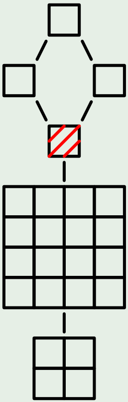
$n \geq 4$



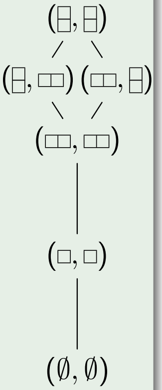
Example (The annihilator in $B_{2,2}(n)$, $n = \dim(V)$)



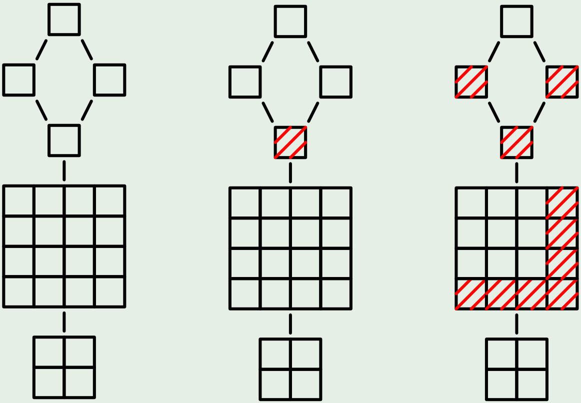
$n \geq 4$



$n = 3$

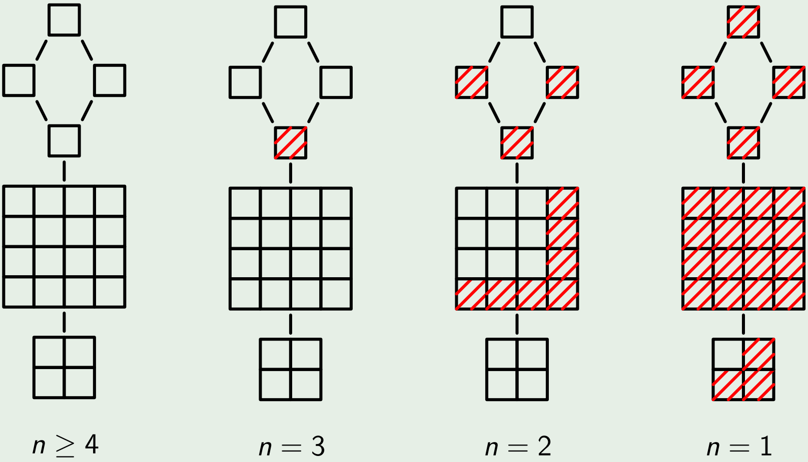


Example (The annihilator in $B_{2,2}(n)$, $n = \dim(V)$)



$n \geq 4$ $n = 3$ $n = 2$

Example (The annihilator in $B_{2,2}(n)$, $n = \dim(V)$)



Filtration

Theorem (Stoll, W)

The mixed tensor space has a $U(\mathfrak{gl}_n)$-$B_{r,s}(n)$-bimodule filtration with factors of the form

$$\text{'dual Weyl module'} \otimes \text{'cell module for } B_{r,s}(n)/\text{annihilator'}.$$