A cell filtration of mixed tensor space, part I

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joint work with Mathias Werth

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Motivation: $R\mathfrak{S}_m$ and tensor space

- R commutative ring with one
- $V = R^n$
- $V^{\otimes m}$ *tensor space*
- $R\mathfrak{S}_m$ acts on the tensor space by permuting components.

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$$m_{\mathfrak{s},\mathfrak{t}} = d(\mathfrak{s})^{-1} y_\lambda d(\mathfrak{t})$$

- The $m_{\mathfrak{s},\mathfrak{t}}$ form a cellular basis, the *Murphy basis* of $R\mathfrak{S}_m$.

Example

$$m = 7,\ \lambda = (3,2,1,1),\ \mathfrak{s} = \begin{array}{|c|c|c|}\hline 1 & 3 & 6 \\ \hline 2 & 5 \\ \cline{1-2} 4 \\ \cline{1-1} 7 \\ \cline{1-1}\end{array},\ \mathfrak{t} = \begin{array}{|c|c|c|}\hline 1 & 2 & 7 \\ \hline 3 & 6 \\ \cline{1-2} 4 \\ \cline{1-1} 5 \\ \cline{1-1}\end{array}.$$

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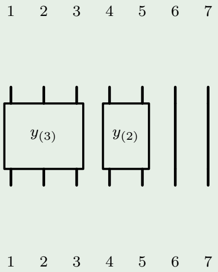

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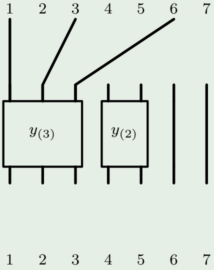

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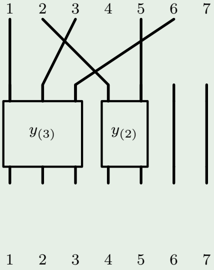

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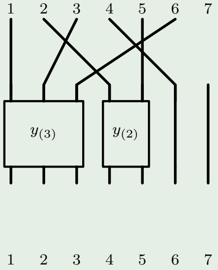

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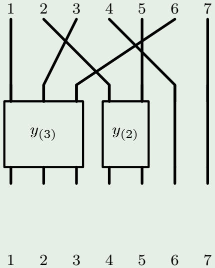

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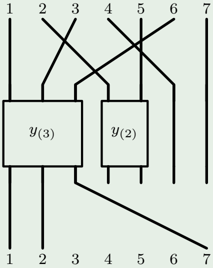

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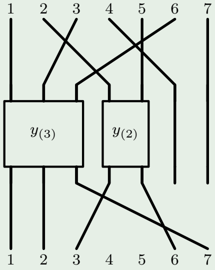

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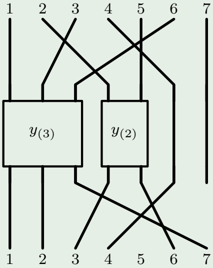

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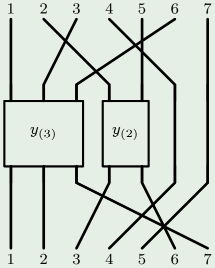

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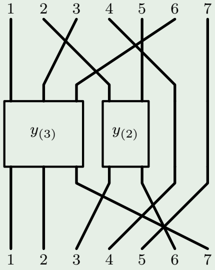

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- The tensor space posseses a filtration with cell modules with respect to the Murphy basis.
- There exists a basis of the tensor space adapted to this filtration.

Properties of the Murphy basis

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- Res C_λ posseses a filtration with cell modules for $R\mathfrak{S}_{m-1}$, the basis $\{c_\mathfrak{s}\}$ is adapted to this filtration.
- the isomorphisms between subquotients and cell modules for the smaller algebra can be described easily using the cell bases.

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$$
C(\trianglerighteq(3,1)) \left\{
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c_{\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4&5\\\cline{1-2}\end{array}} & \mapsto & c_{\begin{array}{|c|c|c|}\hline 1&2&3\\\hline 4\\\cline{1-1}\end{array}} \\
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\end{array}
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$$C(_{\trianglerighteq(2,2)}) \left\{ \begin{array}{lcl} c_{\begin{smallmatrix}1&2&5\\3&4&\end{smallmatrix}} & & \\ c_{\begin{smallmatrix}1&3&5\\2&4&\end{smallmatrix}} & & \\ c_{\begin{smallmatrix}1&2&3\\4&5&\end{smallmatrix}} & \mapsto & c_{\begin{smallmatrix}1&2&3\\4&&\end{smallmatrix}} \\ c_{\begin{smallmatrix}1&2&4\\3&5&\end{smallmatrix}} & \mapsto & c_{\begin{smallmatrix}1&2&4\\3&&\end{smallmatrix}} \\ c_{\begin{smallmatrix}1&3&4\\2&5&\end{smallmatrix}} & \mapsto & c_{\begin{smallmatrix}1&3&4\\2&&\end{smallmatrix}} \end{array} \right.$$

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$$C\big(\trianglerighteq(2,2)\big)/C\big(\trianglerighteq(3,1)\big)\left\{\begin{array}{l} c_{\young(125,34)} + C\big(\trianglerighteq(3,1)\big) \\ c_{\young(135,24)} + C\big(\trianglerighteq(3,1)\big) \end{array}\right.$$

$$\left.\begin{array}{lcl} c_{\young(123,45)} & \mapsto & c_{\young(123,4)} \\ c_{\young(124,35)} & \mapsto & c_{\young(124,3)} \\ c_{\young(134,25)} & \mapsto & c_{\young(134,2)} \end{array}\right\} C_{(3,1)}$$

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  The basis elements corresponding to partitions λ with $\lambda_1 > n$ span the annihilator. (Haerterich)
- $\rightsquigarrow$ $R\mathfrak{S}_m$/annihilator is again a cellular algebra.

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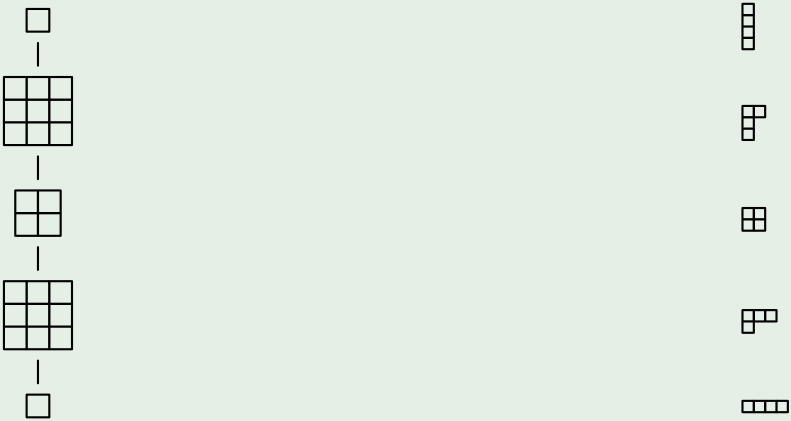

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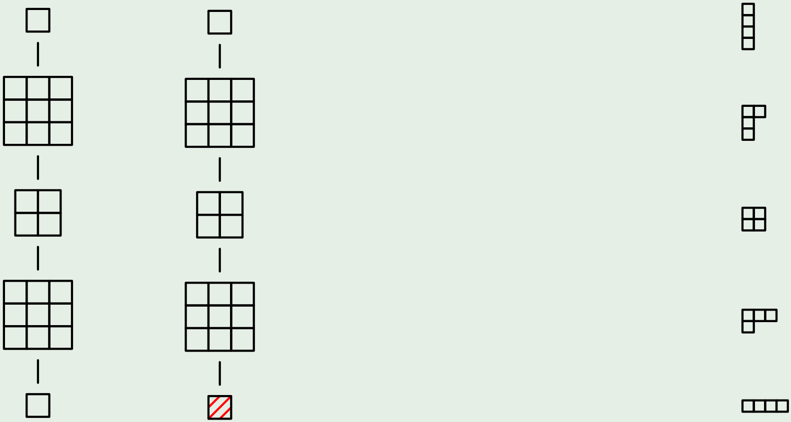

$n \geq 4$ $n = 3$

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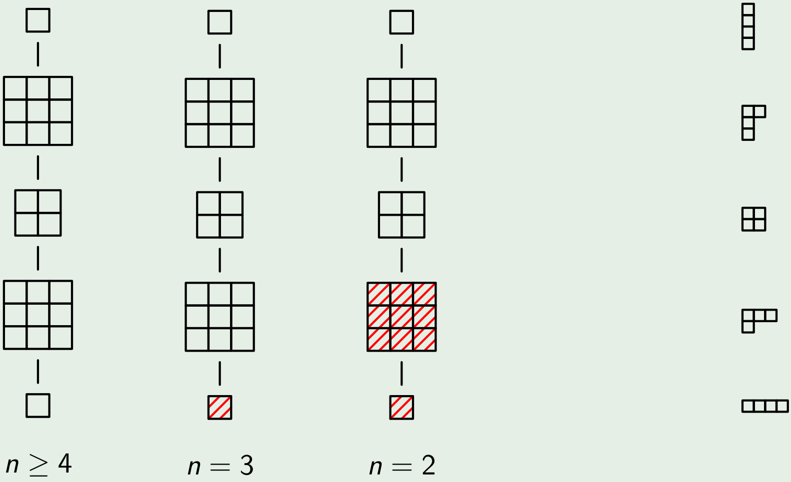

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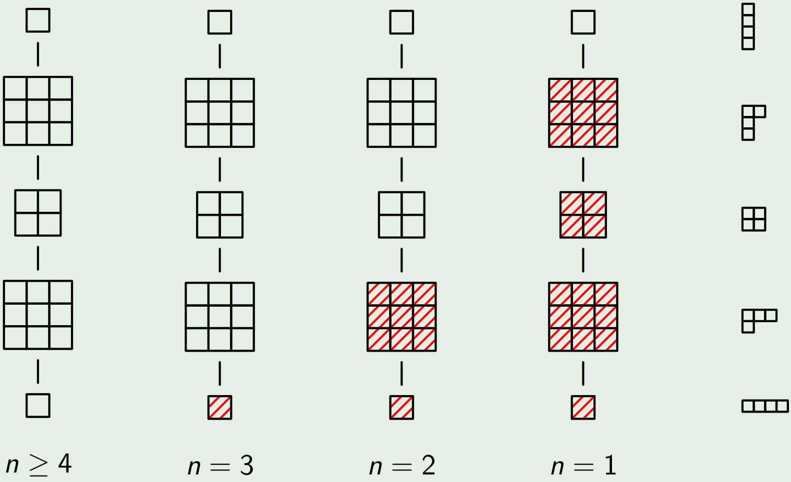

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- Schur-Weyl duality: $\mathrm{End}_{U(\mathfrak{gl}_n)}(V^{\otimes m}) = \mathbb{C}\mathfrak{S}_m/\text{annihilator}$
- $V^{\otimes m} = \oplus_i S_i^{n_i}$,
  S_i pairwise non isomorphic irreducible $U(\mathfrak{gl}_n)$-modules,
  then $\dim_{\mathbb{C}} \mathbb{C}\mathfrak{S}_m/\text{annihilator} = \sum_i n_i^2$

the Littlewood-Richardson rule

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- $V = V(\square)$
- $V(\lambda) \otimes V(\square) = \bigoplus V(\mu)$, the sum over all μ obtained from λ by adding a box in the first n columns.
- $\rightsquigarrow V^{\otimes m}$ can be inductively decomposed into irreducible $U(\mathfrak{gl}_n)$-modules.

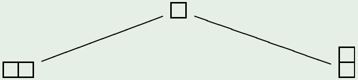
Example ($n \geq 4$)

$V^{\otimes 1}$ $\square$

Example ($n \geq 4$)

$V^{\otimes 1}$

$V^{\otimes 2}$



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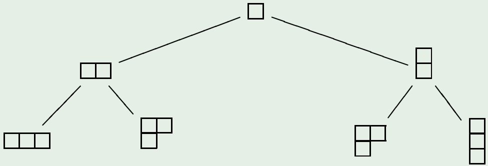

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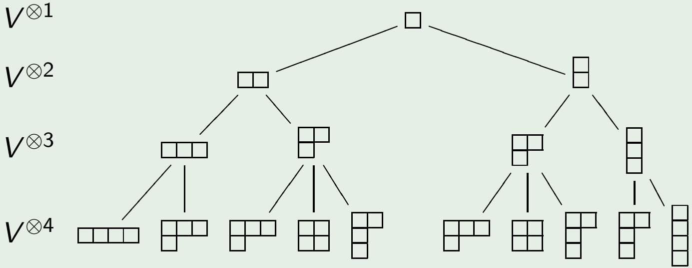

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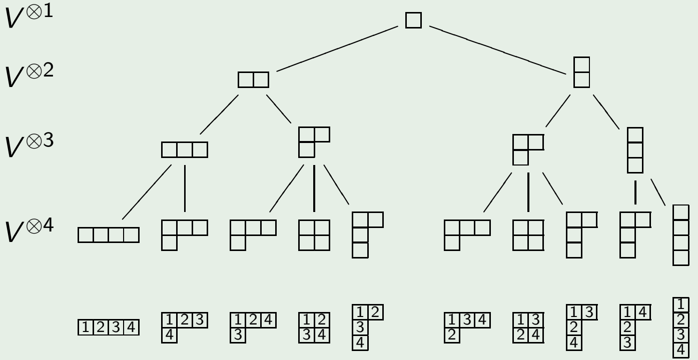

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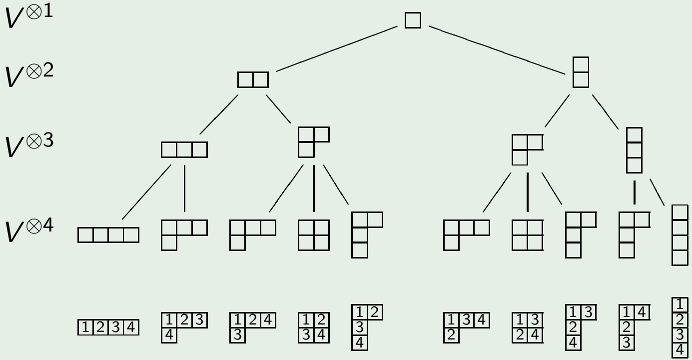

The multiplicity of $V(\lambda)$ in the $U(\mathfrak{gl}_n)$-modules in $V^{\otimes 4}$ is the number of standard λ-tableaux. $\rightsquigarrow$ a basis of $\mathbb{C}\mathfrak{S}_4$ can be indexed by tuples of standard λ-tableaux.

Example ($n = 2$)

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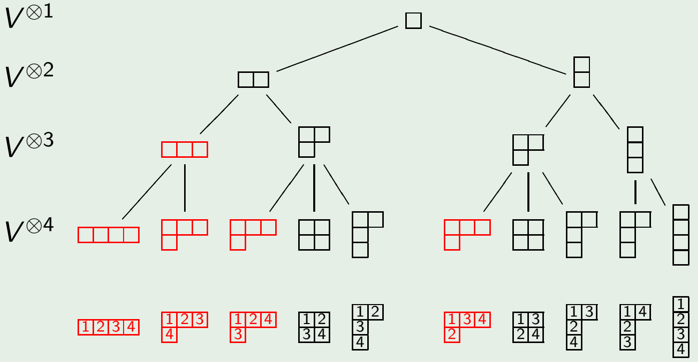

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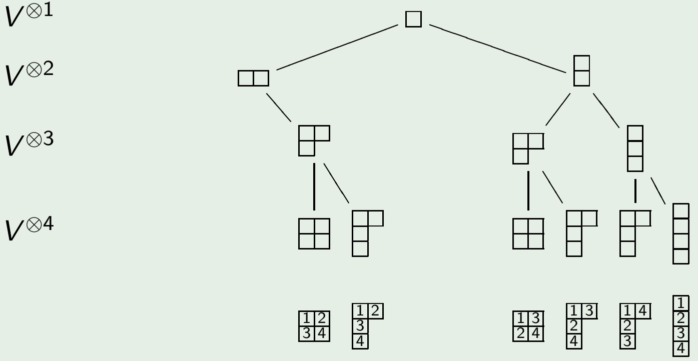

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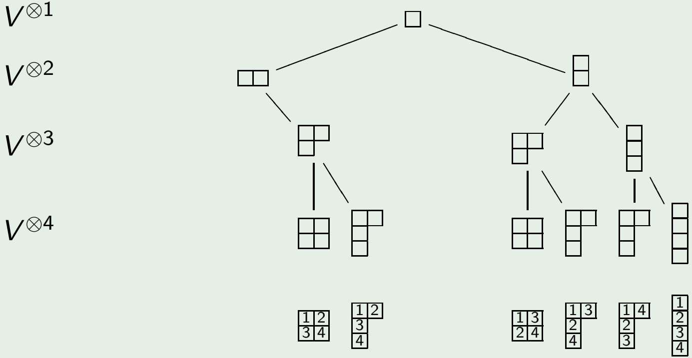

$\rightsquigarrow$ a basis of $\mathbb{C}\mathfrak{S}_4$/annihilator can be indexed by tuples of standard λ-tableaux with $\lambda_1 \leq 2$.

Definition

- A Brauer diagram with $r+s$ vertices in the top and bottom row is called a *walled Brauer diagram*, if
  - all vertical edges do not cross the wall
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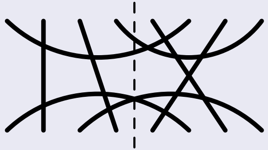

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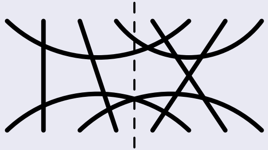

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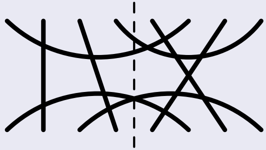

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  Multiplication: Concatenation and deleting closed cycles by multiplication with x

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  $\mathrm{End}_{U(\mathfrak{gl}_n)}(V^{\otimes r} \otimes V^{*\otimes s}) \cong B_{r,s}(n)/\text{annhilator}$
  (Benkart et al., Koike)

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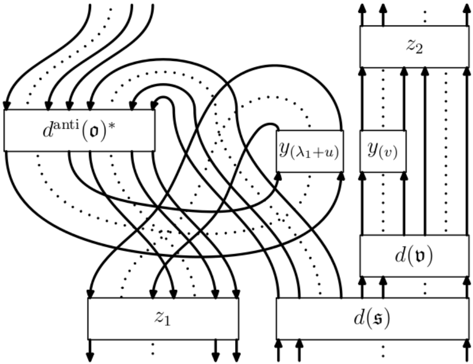
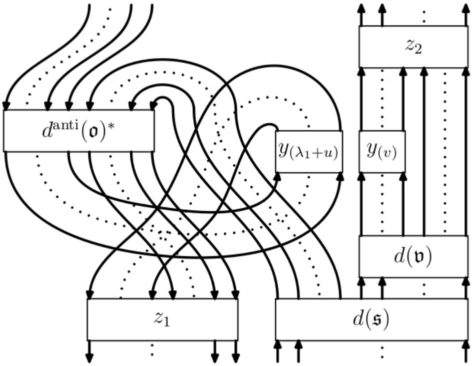
- Even better: We have a solution!



A picture from a proof



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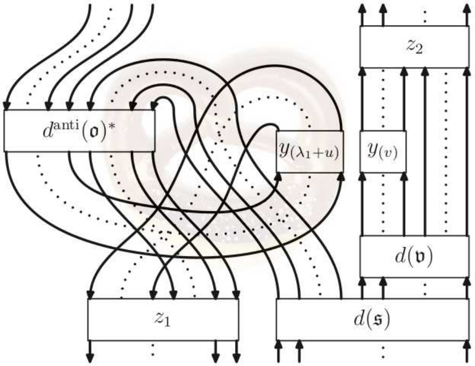

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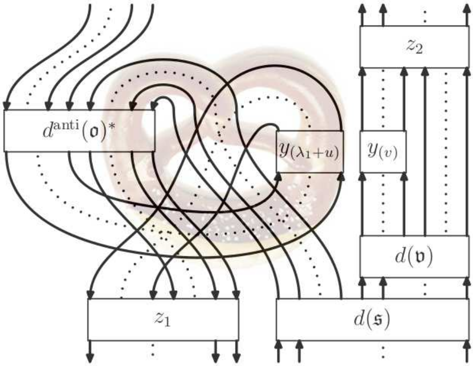

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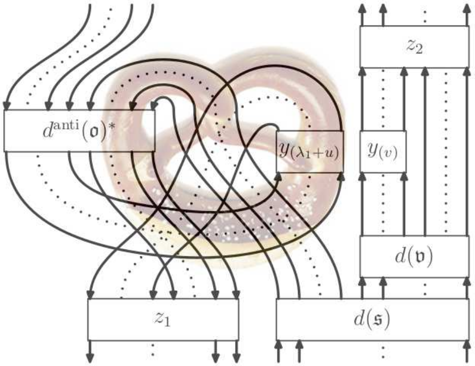

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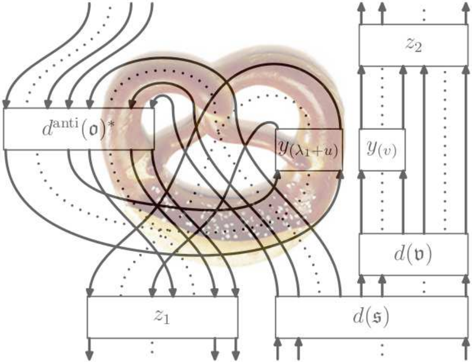

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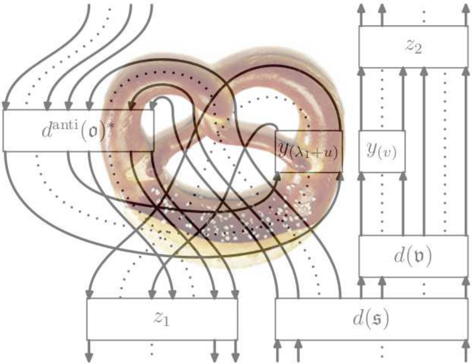

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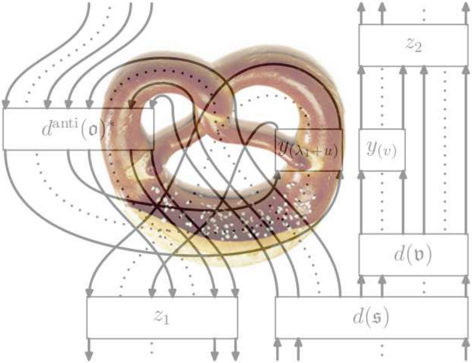

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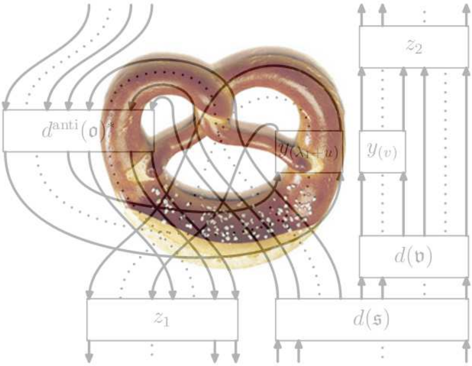

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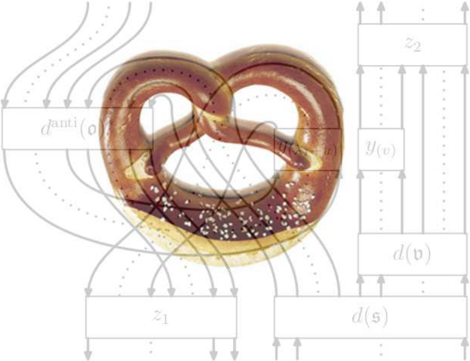

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It's time for a break!