Workshop on Brauer Graph Algebras

Abstracts

Joseph Chuang (City University, London)
Monday 10:40–11:30

Algebras associated to tilings

A certain 'generic' part of the representation theory of symmetric groups in positive characteristic can be modelled by tilings of regions of n-dimensional space by parallelepipeds. For example the tilings tell us about decomposition numbers. Ultimately the goal is to recover the generic representation theory solely from the tilings. The solutions for $n = 1$ and $n = 2$ are given by Brauer tree algebras and by Michael Peach's rhombal algebras, respectively. The correct constructions for arbitrary n could be viewed as higher-dimensional analogues of Brauer tree algebras. This is joint work with Hyohe Miyachi and Kai Meng Tan, and with Will Turner.

Emmanuel Wagner (Université de Bourgogne)
Monday 11:40–12:20

Categorical action of the braid group of the annulus

The usual braid group is an ubiquitous object in mathematics due to its various possible definitions: diagrammatic presentation, mapping class group , fundamental group of configuration space ... Using all these points of view Khovanov and Seidel constucted a faithful categorical action of the braid group.The usual braid group is also called the Artin group of type A. Among all other Artin groups of finite type the one that shares very similar various possible definitions is the Artin group of type B. Using a similar approach to Khovanov-Seidel, we construct a categorical action of the Artin group of type B which is a categorification of a natural homological representation. This a joint work with A. Gadbled and A-L. Thiel.

Dusko Bogdanic (Universität Graz)
Monday 15:00–15:50

Gradings on Brauer tree algebras

In this talk we study non-trivial gradings on finite dimensional associative algebras, and in particular, we apply some of our general results to the case of Brauer tree algebras. We construct non-negative gradings on a basic Brauer tree algebra A_Γ corresponding to an arbitrary Brauer tree Γ of type (m, e). We do this by transferring gradings via derived equivalence from a basic Brauer tree algebra A_S, whose tree is a star with the exceptional vertex in the middle, to A_Γ. The grading on A_S comes from the tight grading given by the radical filtration. We also prove that there is unique grading on A_Γ up to graded Morita equivalence and rescaling.

Hideto Asashiba (Shizuoka University)

Monday 16:00–16:40

The smash product of Brauer graphs with group weights and Brauer permutations

For a group G (not necessarily abelian or finite) and a Brauer graph with an admissible G-weight we define their smash product, which is again a Brauer graph and is a covering of the original one with group G, and introduce a new concept of a Brauer permutation that gives another presentation of a Brauer graph and is convenient for describing the smash product and also Brauer quivers. This construction is compatible with the smash product construction of G and a G-graded linear category, and in particular when G is a finite abelian group, it gives simple descriptions of coverings of Brauer graphs investigated by Green-Schroll-Snashall.

Rachel Taillefer (Université Blaise Pascal, Clermont Ferrand II)

Monday 17:20–18:10

On a generalisation of N-Koszul algebras for Brauer graph algebras

Koszul algebras are a well-known and much studied class of algebras. They were generalised in 2001 by Roland Berger to N-Koszul algebras. This means that if we write the algebra as a quotient of a tensor algebra $A = T_k(V)/I$, the ideal I can be generated by elements of degree N and that the projective modules in a minimal graded projective resolution of k can be generated in specific degrees depending on N. Moreover, the Ext algebra of k is generated in degrees 0, 1 and 2. This notion has been generalised since in several ways. We are interested in two of them: - an algebra is called $\mathcal{K}_2$ if it is graded and if its Ext algebra is generated in degrees 0, 1 and 2 [Cassidy-Shelton]; - an algebra $A = T_k(V)/I$ is called 2-d-determined if the ideal I can be generated by elements of degrees 2 and d, where $d > 2$ is an integer, and the projective modules in a minimal graded projective resolution of k can be generated in specific degrees depending on 2 and d [Green-Marcos]. The aim of this talk is to give examples of such algebras, within the class of Brauer graph algebras, and to compare $\mathcal{K}_2$ Brauer graph algebras and 2-d-determined Brauer graph algebras.

This is joint work with E.L. Green, S. Schroll and N. Snashall.

Joseph Grant (University of East Anglia, Norwich)

Tuesday 9:00–9:50

A generalization of certain Brauer tree algebras

Brauer tree algebras appear in different places, so if we hope to generalize them we need to choose which properties are important to us. Indecomposable projectives of Brauer tree algebras (without multiplicity) are spherical, and these algebras encode relations between spherical twists, including the classical braid relations. In this talk we will consider a new class of algebras which generalize certain Brauer tree algebras from this perspective. Parts of this work are joint with Osamu Iyama.

Takuma Aihara (Tokyo Gakugei University)

Tuesday 10:00–10:40

Tilting mutation and flip of Brauer graph algebras

The notion of mutation plays a crucial role in representation theory of algebras. In particular, tilting mutation - the process which constructs a new tilting object from a given one - gives a systematic approach to study tilting theory. A natural problem is to explicitly describe the endomorphism algebra of a tilting object obtained by tilting mutation. In this talk, we give an answer to this problem for irreducible mutations of tilting complexes of Brauer graph algebras.

Alexandra Zvonareva (Saint Petersburg State University)

Tuesday 11:20–12:10

On stably biserial algebras

By a result of Pogorzaly stably biserial algebras are algebras potentially stably equivalent to self-injective special biserial algebras. By an example of Ariki, Iijima and Park the classes of stably biserial and selfinjective special biserial algebras do not coincide. We will analyze the structure of symmetric stably biserial algebras and computing Külshammer ideals show that these algebras can be derived equivalent to symmetric special biserial (Brauer graph) algebras if and only if they are symmetric special biserial themselves. If time permits, I will also discuss some generalizations of Külshammer ideals.

Robert Marsh (University of Leeds)

Tuesday 15:00–15:50

Brauer graph algebras and cluster mutations

By interpreting a Brauer graph as a ribbon graph, we show that a Brauer graph algebra, unique up to derived equivalence, can be associated to a compact oriented surface with marked points. In the case of a disc with marked points, we give a a dual version of this statement: the rotation of a diagonal in an m-angulation gives rise to a Whitehead move in the dual graph, and we construct an explicit tilting complex on the corresponding Brauer graph algebras which corresponds to this move.

Aaron Chan (Uppsala Universitet)

Tuesday 16:00–16:40

Brauer graph algebras via surface combinatorics I

Given a Riemann surface, one can construct various triangulated categories. These are usually motivated by the homological mirror symmetry conjecture and related Calabi-Yau algebras/categories. In these constructions, curves on Riemann surfaces often give rise to an important class of objects. Since a Brauer graph can be embedded into a Riemann surface with marked points and boundary, one can also ask if the curves of such a surface give us any interesting objects in the triangulated categories associated to a Brauer graph algebra. Generalising ideas from works by Khovanov-Seidel/Seidel-Thomas and Marsh-Schroll, we associate curves on Riemann surface with complexes in the bounded homotopy category. In this talk, we will give details on this construction and show

how one can interpret some homological phenomenons and problems of a Brauer graph algebra using the combinatorics of the associated surface. This is joint work with Takahide Adachi.

Laurent Demonet (Nagoya University)
Tuesday 17:20–18:10
Algebras of partial triangulations
This is a report on [Dem16].

We introduce a class of finite dimensional algebras coming from partial triangulations of marked surfaces. A partial triangulation is a subset of a triangulation.

This class contains Jacobian algebras of triangulations of marked surfaces [LF09] (see also [DWZ08]) and Brauer graph algebras [WW85]. We generalize properties which are known or partially known for Brauer graph algebras and Jacobian algebras of marked surfaces. In particular, these algebras are symmetric when the considered surface has no boundary, they are at most tame, and we give a combinatorial generalization of flips or Kauer moves on partial triangulations which induces (in most cases) derived equivalences between the corresponding algebras. Notice that we also give an explicit formula for the dimension of the algebra.

REFERENCES

[Dem16] Laurent Demonet. Algebras of partial triangulations. arXiv: 1602.01592, 2016.

[DWZ08] Harm Derksen, Jerzy Weyman, and Andrei Zelevinsky. Quivers with potentials and their representations. I. Mutations. *Selecta Math. (N.S.)*, 14(1):59–119, 2008.

[LF09] Daniel Labardini-Fragoso. Quivers with potentials associated to triangulated surfaces. *Proc. Lond. Math. Soc. (3)*, 98(3):797–839, 2009.

[WW85] Burkhard Wald and Josef Waschbüsch. Tame biserial algebras. *J. Algebra*, 95(2):480–500, 1985.

Alexander Zimmermann (Université de Picardie, Amiens)
Wednesday 9:00–9:50
Batalin-Vilkovisky structure on the Hochschild cohomology of Frobenius algebras

This is joint work with Thierry Lambre and Guodong Zhou. If there is a differential Δ on the Hochschild cohomology of an associative algebra A such that the Gerstenhaber Lie bracket on the Hochschild cohomology of A is the obstruction of Δ to be a derivation with respect to cup product, then we say that the Hochschild cohomology of A has a Batalin-Vilkovisky structure. Kowalzig and KrÃd'hmer showed that the Hochschild cohomology of symmetric algebras has a Batalin-Vilkovisky structure. We prove that also the Hochschild cohomology of a Frobenius algebra with diagonalisable Frobenius automorphism is Batalin-Vilkovisky. We give a number of practical criteria to verify this diagonalisability condition, and prove that they are satisfied for certain special biserial algebras.

Drew Duffield (University of Leicester)

Wednesday 10:00–10:40

Auslander-Reiten components of Brauer graph algebras

We provide an algorithm for constructing the stable Auslander-Reiten component containing a given string module of a symmetric special biserial algebra using only information from its underlying Brauer graph. We also show that the structure of the Auslander-Reiten quiver is closely related to the distinct Green walks around the Brauer graph and detail the relationship between the precise shape of the stable Auslander-Reiten components for domestic Brauer graph algebras and their underlying graph. Furthermore, we show that the specific component containing a given simple or indecomposable projective module for any Brauer graph algebra is determined by the edge in the Brauer graph associated to the module.

Sibylle Schroll (University of Leicester)

Wednesday 11:20–12:10

A generalization of Brauer graph algebras

This talk is concerned with special multiserial algebras. Special multiserial algebras are a generalisation of special biserial algebras. The connection with Brauer graph algebras stems from the fact that the class of symmetric special biserial algebras coincides with the class of Brauer graph algebras. We start by showing general properties of special multiserial algebras. We then introduce a generalization of Brauer graph algebras, called Brauer configuration algebras, and we show that they coincide with symmetric special multiserial algebras.

Andrzej Skowroński (Nicolaus Copernicus University, Toruń)

Wednesday 14:30–15:20

Weighted surface algebras

Report on joint work with Karin Erdmann.

From the Tame and Wild Theorem of Drozd the class of finite dimensional algebras over an algebraically closed field K may be divided into two disjoint classes. The first class consists of the tame algebras for which, in each dimension d, all but finitely many isomorphism classes of d-dimensional indecomposable modules come in a finite number of one-parameter families. The second class is formed by the wild algebras whose representation theory comprises the representation theories of all finite dimensional algebras over K. Accordingly, we may hope to classify the indecomposable finite dimensional modules only for the tame algebras. The representation theory of arbitrary tame algebras is still only emerging. One of the interesting open problems concerns the classification of the Morita equivalence classes of all tame symmetric finite dimensional algebras.

The aim of the talk is to introduce the weighted surface algebras of triangulated surfaces with arbitrarily oriented triangles and describe their basic properties. It is expected that the orbit closures of the weighted surface algebras contain algebras playing a prominent role in the classification of the Morita equivalence classes of the tame symmetric algebras of generalized dihedral, generalized semidihedral, and generalized quaternion types.

Takahide Adachi (Nagoya University)
Wednesday 15:30–16:10

Brauer graph algebras via surface combinatorics II

This is the continuation of talk by Aaron Chan. In this talk, we will present some properties of the complexes associated to curves on surface presented in the previous talk. Then we will eventually focus on the two-term complexes and explain some applications. This talk contains joint works with Takuma Aihara and Aaron Chan.

Karin Erdmann (University of Oxford)
Wednesday 16:50–17:40

Algebras of generalised quaternion type

Assume A is a finite-dimensional indecomposable algebra over some field K. We say that A is of generalised quaternion type if A is tame, symmetric, and all simple A-modules have Ω-period four. If in addition the Cartan matrix of A is non-singular, then A is an algebra of quaternion type which was studied earlier, in the context blocks of finite groups with quaternion defect groups.

In this lecture, we discuss the classification of algebras of generalised quaternion type when the quiver of the algebra is 2-regular (that is, there are precisely two arrows ending and two arrows starting at each vertex of the quiver). Any such algebra with at least three simple modules is Morita equivalent to a weighted surface algebra. We also explain how partial degenerations, and degenerations lead naturally to algebras which generalise algebras of semidihedral type, and algebras of dihedral type which were introduced in the context of classifying tame blocks.