Extendable t-structure and finitistic dimension of small triangulated categories

Junhua Zheng

(joint work with R. Biswas, H. X. Chen, K. M. Rahul, and C. J. Parker)

Abstract

A **good metric** $\mathcal{M} = \{\mathcal{M}_n\}_N$ on a triangulated category $(\mathcal{S}, \Sigma)$ is a sequence of additive extension-closed subcategories such that $\Sigma^i \mathcal{M}_{n+1} \subseteq \mathcal{M}_n \subseteq \mathcal{S}$, for all $n \in N$ and $i \in \{-1, 0, 1\}$. Whenever $\mathcal{S}$ is small, Neeman has recently constructed, for any good metric $\mathcal{M}$, a new small triangulated category $S_{\mathcal{M}}(\mathcal{S})$ called the **completion** of $\mathcal{S}$ (relative to $\mathcal{M}$), as a subcategory of $\text{Mod-}\mathcal{S} := [\mathcal{S}^{\text{op}}, \text{Ab}]$.

Starting from the observation that the assignment $\mathcal{S} \mapsto S_{\mathcal{M}}(\mathcal{S})$ extends to a correspondence taking each subcategory $\mathcal{X} \subseteq \mathcal{S}$ to a suitable $S_{\mathcal{M}}(\mathcal{X}) \subseteq S_{\mathcal{M}}(\mathcal{S})$, we will introduce a class of t-structures $t = (\mathcal{D}^0, \mathcal{D}^1)$ on $\mathcal{S}$, called **extendable** (relative to $\mathcal{M}$), for which $S_{\mathcal{M}}(t) := (S_{\mathcal{M}}(\mathcal{D}^0), S_{\mathcal{M}}(\mathcal{D}^1))$ is a t-structure on $S_{\mathcal{M}}(\mathcal{S})$. We will then show that, in this case, the heart of $S_{\mathcal{M}}(t)$ is always equivalent to the one of t, and that $S_{\mathcal{M}}(t)$ is bounded above, if so is t.

In the second part of the talk, after recalling a construction by Neeman that associates to any object $G \in \mathcal{S}$ a suitable good metric $\mathcal{M}_G$, we will concentrate on the new notion of **finitistic dimension** $\text{findim}(\mathcal{T}, H)$ of a small triangulated category $\mathcal{T}$ at the object $H \in \mathcal{T}$, also comparing it to related invariants. Combining all these pieces, we will show that any bounded t-structures on $\mathcal{S}$ is extendable relative to $\mathcal{M}_G$, provided $\text{findim}(\mathcal{S}^{\text{op}}, G) < \infty$. As a further application of the theory we will obtain that: If there is $G \in \mathcal{S}$ such that $\text{findim}(\mathcal{S}^{op}, G) < \infty$ then: either $\mathcal{S}$ admits no bounded t-structure or, when it does, they all are equivalent.

Junhua Zheng, University of Stuttgart

E-mail address: Zheng.Junhua@mathematik.uni-stuttgart.de.