

## SIMPLY-CONNECTED GRADINGS AND INTRINSIC FUNDAMENTAL GROUPS OF MATRIX ALGEBRAS

ABSTRACT. Simply-connected gradings of a linear category  $\mathcal{B}$ , i.e. the maximal connected gradings with respect to quotient maps, are key in computing the intrinsic fundamental group  $\pi_1(\mathcal{B})$ . Our main concern here is the 1-object linear category  $\mathcal{B} = M_n(\mathbb{C})$ . Its strongly-connected gradings turn out to be in one-to-one correspondence with  $\text{Aut}(G)$ -orbits of non-degenerate classes in  $H^2(G, \mathbb{C}^*)$ , where  $G$  are all groups of central type whose orders divide  $n^2$ . We classify the groups of central type of order  $n^2$  where  $n$  is a square-free number and compute the family  $\Lambda$  of positive integers (containing all prime numbers)  $n$  such that there is a unique group of central type of order  $n^2$ , namely  $C_n \times C_n$ . By determining the simply-connected gradings of  $M_n(\mathbb{C})$ , we establish some results concerning  $\pi_1(M_n(\mathbb{C}))$  for every  $n \in \Lambda$ . We push this investigation further and show how dimensions of complex projective representations determine the simply-connected gradings of any finite dimensional semi-simple complex algebras.