

Aspects of representation-finiteness

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Fourty years ago, Auslander and Ringel-Tachikawa have shown that an artinian algebra is representation-finite if and only if all modules decompose into finitely generated ones. Artinian algebras are noetherian rings Λ of Krull dimension zero. If the dimension is raised, and Λ is not too far from commutative, Auslander proved, one decade later, that representation-finiteness forces Λ to be an isolated singularity, or equivalently, that almost split sequences exist for the representations over Λ . Auslander's splitting-big-objects theorem has never been extended to dimensions > 1 , but analogies, beyond dimensional restriction, were found among Gorenstein-projectives by X.-W. Chen (AIM, 2008) and Beligiannis (AIM, 2011).

Using the interplay between 2- and 3-termed complexes in homotopy categories, we sketch a general perspective to close the gaps between those aspects of representation-finiteness. The question "What is a big object?" necessarily occurs in such a context. For the basic finiteness of indecomposables criterion, the main gadget is a functorial Auslander-Reiten theory of morphisms instead of objects. In dimension ≤ 1 , it has been used for the characterization of finite Auslander-Reiten quivers and a construction of big indecomposables.