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Abstract

Irreducible constituents of minimal degree in super characters of the finite unitriangular groups

Let q be a prime power n a natural number and U the group of lower unitriangular $n \times n$ -matrices with entries in the field F with q elements. It is known that determining the conjugacy classes of U for all n and q is a wild problem. Even finding their number as function $C(q,n)$ of q and n and hence the number of distinct irreducible complex characters is still an open problem. A longstanding conjecture, contributed to G. Higman states, that $C(q,n)$ should be a polynomial with integral coefficients depending only on n , evaluated at q . The super character theory of Andrè and Yan provides an approximation to the problem of classifying the irreducible complex U -characters. Super characters are certain pairwise orthogonal characters of U , which can be described combinatorially, such that every irreducible character of U occurs in precisely one super character as constituent. There is an also combinatorially determined lower bound (a power of q) for the degree of an irreducible constituent occurring in a super character, called minimal degree. We give a necessary and sufficient condition for a super character to have constituents whose degree assumes this lower bound, and construct these constituents. Moreover we show, that the number of distinct irreducible U -characters, which have minimal degree in their super characters, is a polynomial in $q-1$ with nonnegative coefficients, which depends only on n .