

CONVEX GEOMETRY FOR TRIANGULATED CATEGORIES

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Abstract.

Fans and other convex-geometric objects have recently appeared in homological algebra in several related contexts. For example, as g -fans in the silting theory of finite-dimensional algebras and as scattering diagrams in Bridgeland stability theory. I will discuss joint work-in-progress with David Pauksztello, David Ploog and Jon Woolf on a general construction which we hope will provide a natural and unifying framework. Starting with a triangulated category D and a finite rank quotient lattice L of its Grothendieck group, we show that each heart H in D determines a closed convex 'heart cone' in the dual vector space $V = \text{Hom}(L, R)$. The heart cones of H and all its forward tilts form a 'heart fan' in V . If H is 'algebraic', i.e. is a length category with finitely many simple objects, then the heart cone is simplicial and the heart fan is complete. The heart fans for all hearts in D can be assembled into a 'multifan'. Under certain assumptions, the tangent space of this can be interpreted as a space of 'lax stability functions' on D and the complex manifold $\text{Stab}(D)$ of Bridgeland stability conditions embeds as an open subset.