

Representation theory and knot invariants - exercises

- (1) (a) Is the adjoint representation V of $\mathfrak{sl}(2, \mathbb{C})$ simple?
(b) Decompose the tensor product $V \otimes V$ into a direct sum of simple representations.
- (2) Are there zero divisors in universal enveloping algebras?
- (3) Let U_q be the quantised universal enveloping algebra of $\mathfrak{sl}(2, \mathbb{C})$. Determine the simple Verma modules over U_q .
- (4) Let U_q be the quantised universal enveloping algebra of $\mathfrak{sl}(2, \mathbb{C})$.
 - (a) Determine all grouplike elements in U_q .
 - (b) Determine all elements $x \in U_q$ such that $\Delta(x) = 1 \otimes x + x \otimes K^{-1}$.
- (5) Let U_q be the quantised universal enveloping algebra of $\mathfrak{sl}(2, \mathbb{C})$. Let $n \in \mathbb{N}_0$ and $\epsilon \in \{+, -\}$. Is the simple U_q -module $V_{\epsilon, n}$ isomorphic to its dual?

This will be discussed in the problem class on Friday, 19th of July.

Homepage of the course:

<http://www.iaz.uni-stuttgart.de/LstAGeoAlg/Koenig/knotsreps/Knotsreps.html>