Problem sheet 2 Thursday, 2nd of May, 2019

Representation theory and knot invariants - exercises

(1) Let π be the group homomorphism $B_n \to \Sigma_n$ given in the lectures. Let $\beta \in B_n$ be a braid and $\hat{\beta}$ its closure. Determine the number of connected components of $\hat{\beta}$ in terms of the permutation $\pi(\beta)$.

(2) Lay a braid out flat and consider it as a bowling alley with n lanes, the lanes going over each other according to the braid. Assume that at each crossing the ball has probability t of falling off the lane and continuing in the lane below. Given i and j, what is the probability that a ball bowled in the ith lane ends up in the jth?

(3) Let R be a commutative ring (with unit) and $a_1, \dots, a_n$ a finite set of elements in R. Show that the following conditions are equivalent:
   (a) R is the ideal $(a_1, \dots, a_n) \subset R$ generated by the elements $a_1, \dots a_n$, and moreover for all $i \neq j$, the product $a_i a_j$ is nilpotent.
   (b) There exists a natural number k such that R decomposes as $(a_1^k) \oplus \cdots \oplus (a_n^k)$.
   (c) There exists a finite subset $\{b_1, \dots, b_n\}$ of R such that $\sum_{i=1}^n a_i b_i = 1$ and moreover for all $i \neq j$ the product $a_i b_j$ is nilpotent.

This will be discussed in the problem class on Thursday, 9th of May.

Homepage of the course:
`http://www.iaz.uni-stuttgart.de/LstAGeoAlg/Koenig/knotsreps/Knotsreps.html`