(1) Let $\pi$ be the group homomorphism $B_n \to \Sigma_n$ given in the lectures. Let $\beta \in B_n$ be a braid and $\hat{\beta}$ its closure. Determine the number of connected components of $\hat{\beta}$ in terms of the permutation $\pi(\beta)$.

(2) Lay a braid out flat and consider it as a bowling alley with $n$ lanes, the lanes going over each other according to the braid. Assume that at each crossing the ball has probability $t$ of falling off the lane and continuing in the lane below. Given $i$ and $j$, what is the probability that a ball bowled in the $i$th lane ends up in the $j$th?

(3) Let $R$ be a commutative ring (with unit) and $a_1, \ldots, a_n$ a finite set of elements in $R$. Show that the following conditions are equivalent:

(a) $R$ is the ideal $(a_1, \ldots, a_n) \subset R$ generated by the elements $a_1, \ldots, a_n$, and moreover for all $i \neq j$, the product $a_ia_j$ is nilpotent.

(b) There exists a natural number $k$ such that $R$ decomposes as $(a_1^k) \oplus \cdots \oplus (a_n^k)$.

(c) There exists a finite subset $\{b_1, \ldots, b_n\}$ of $R$ such that $\sum_{i=1}^n a_ib_i = 1$ and moreover for all $i \neq j$ the product $a_ib_j$ is nilpotent.

This will be discussed in the problem class on Thursday, 9th of May.

Homepage of the course:
http://www.iaz.uni-stuttgart.de/LstAGeoAlg/Koenig/knotsreps/Knotsreps.html