Preview

When an algebra A is given, one would like to know, for instance, the number of simple A-modules, up to isomorphism / the number of indecomposable modules, up to isomorphism (if it is finite) / the global dimension / the Ext-groups between simples, etc.
Solving these problems for a particular A is nice, but it would be even nicer to solve them for a whole class of ~~modules~~ algebras, including A, at the same time.
Morita equivalence provides one way to find such a class of algebras: Every algebra B Morita equivalent to A has the same number of simples / of indecomposables / the same global dimension / the same Ext-groups between simples, etc.
Isomorphic algebras always are Morita equivalent, but Morita equivalence goes far beyond that and does, for example, not preserve the k-dimension.

So, what is Morita equivalence? We require A-Mod and B-Mod to be equivalent as categories. Is this enough? Yes, in the first part of this chapter we will show that such an equivalence automatically sends exact sequences to exact sequences, simple modules to simple modules, projective modules to projective modules, that it preserves the abelian group structure of homomorphism sets, etc.
This implies that all the data above, and many more, are preserved under such equivalences of categories. A and B then are called Morita equivalent.

The main result, Morita's theorem, then tells us precisely when two algebras A and B are Morita equivalent.

For instance, the field k is Morita equivalent to the k-algebra $Mat(n\times n, k)$, for each $n \in \mathbb{N}$. More interestingly, every finite dimensional k-algebra over an algebraically closed field k is Morita equivalent to a bound quiver algebra (which explains why these are among our favourite examples).

The equivalence $A\text{-}Mod \simeq B\text{-}Mod$ in Morita's theorem is a Hom-functor (it also can be written as a tensor product functor). The final theorem in this chapter will clarify the special role of tensor product (or Hom) functors, making them unique.