Preview

In chapter 1, we have looked at short exact sequences and extensions of modules. Ext^1 collects equivalence classes of extensions and it turned out to be an abelian group. We also have seen that projective and injective modules play a special role: $Ext^1(P,-)=0$ for P projective and $Ext^1(-,I)=0$ for I injective.
In chapter 3, we have identified Ext^1 with a quotient of a particular Hom, and we have seen that Ext^1 repairs in some sense the lack of exactness of Hom. Theorems 3.3, 3.9 and 3.10 make that precise – and raise the question how to continue after Ext^1: $Ext^2, Ext^3, \ldots$?
To define Ext^n and put it into a general picture is now our main goal.
We have already seen two important ingredients of the general picture: Every module has a (in fact: many) projective resolution and a (in fact: many) injective resolutions and is therefore related with modules that behave particularly well in terms of Ext^1.
In chapter 5 we have seen that Hom and Ext are functors, which allows to study them globally for all modules, not just in particular cases.

Chapter 6 will define Ext^n in full generality, in a general categorical context.
This chapter has three parts:
The first part introduces complexes; these generalise modules (complexes with one non-zero term), short exact sequences, long exact sequences, but also include sequences that are not exact, but could be made ~~e~~ exact by adding terms as in 3.3, 3.9 or 3.10. A category of complexes will be defined and a functor called homology will get introduced to measure lack of exactness. Homology (and a variation called cohomology) will be shown always to yield a long exact sequence.

The second part looks at projective and injective resolutions (which are complexes, too). How can we compare different resolutions of the same module? How can we construct new resolutions from known ones? For instance, given a short exact sequence $0 \to X \to Y \to Z \to 0$ and projective resolutions of X and of Z, how can we systematically construct a projective resolution of the middle term Y?
After the second part there will be another preview, putting the material into the context needed in the third part.
The third part constructs functors in a very general and perhaps frightening abstract way, which gives all the properties we want – abelian group structure, functoriality, long exact sequences – in one go. Not only for Ext^n, but also in an equally important situation to be studied afterwards (replacing Hom by tensor product). The construction also explains that the construction of Ext^n is best possible, in the sense of a universality property.
The construction of Ext^n comes with a recipe, which will turn out to be unexpectedly easy to carry out and to use.