Exercises on complexes

(1) Let S be a set, $S \neq \emptyset$. For $i \geq 0$ let E_i be the free $\mathbb{Z}$-module with basis all $(i+1)$-tuples $(x_0, \dots, x_i)$ of elements of S.
Define $\varepsilon = d_0 : E_0 \to \mathbb{Z}$ the $\mathbb{Z}$-module homomorphism sending a basis element x_0 to ~~@~~ 1 ("augmentation")
and for $i \geq 0$, $d_{i+1} : E_{i+1} \to E_i$ by

$$(x_0, \dots, x_{i+1}) \longmapsto \sum_{j=0}^{i+1} (-1)^j (x_0, \dots, \underset{\uparrow \text{omitted}}{\hat{x}_j}, \dots, x_{i+1})$$

(a) Show $d \circ d = 0$.
(b) Fix $z \in S$ and define $h : E^i \to E^{i+1}$ by $(x_0, \dots, x_i) \longmapsto (z, x_0, \dots, x_i)$.
Prove that $d \circ h + h \circ d = \mathrm{id}_{E}$, where E_*: $\dots \to E_{i+1} \to E_i \to \dots \to E_0 \xrightarrow{\varepsilon} \mathbb{Z} \to 0$
(c) Prove that E_* is an ex~~t~~act chain complex, i.e. its homology vanishes.

(2) Let G be a group and $S = G$. G acts on E_i by $g \cdot (g_0, \dots, g_i) := (g g_0, \dots, g g_i)$.
Let $R = \mathbb{Z}G$ be the group algebra over the integers.
Let $\mathbb{Z}$ be the trivial R-module: $g \cdot n := n$ $\forall g \in G, n \in \mathbb{Z}$.
Show that E_* provides a free resolution of $\mathbb{Z}$ over R, i.e. all E_i are free R-modules.

(When M is a $\mathbb{Z}G$-module, applying $\mathrm{Hom}_{\mathbb{Z}G}(-, M)$ to E_* yields a cochain complex, where one drops the term $\mathrm{Hom}_{\mathbb{Z}G}(\mathbb{Z}, M)$:
$0 \to \mathrm{Hom}_{\mathbb{Z}G}(E_0, M) \to \mathrm{Hom}_{\mathbb{Z}G}(E_1, M) \to \mathrm{Hom}_{\mathbb{Z}G}(E_2, M) \to \dots$
Its cohomology $H^*(\mathrm{Hom}_{\mathbb{Z}G}(E_*, M)) =: H^*(G, M)$ is called group cohomology. This is ~~the~~ a subject of its own, used in group theory, algebraic topology and elsewhere. Extensions of groups can be studied by group cohomology, but this is more complicated than Ext^1 for modules — groups are not modules over anything.)
Note: $\mathbb{Z}$ could be replaced by a field k.