

Preview

Our subject is representation theory of algebras. Our main tool, $\text{Ext}_A^1(X, Y)$ for A -module X and Y , is taken from homological algebra - an area that provides widely applicable tools and that is closely connected with category theory.

"Category theory takes a bird's eye view of mathematics. From high in the sky, details become invisible, but we can spot patterns that were impossible to detect from ground level." (From the introduction to Tom Leinster's book *Basic Category Theory* - freely available on

<https://arxiv.org/abs/16.12.09375>)

We have already used some language of category theory, in particular universal properties, when defining kernels and cokernels, pullbacks and pushouts, etc. In this section we will define categories in general and identify properties satisfied by module categories, but also by categories in topology and geometry and elsewhere.

We will define functors, which will help us to understand what $\text{Ext}^1(-, -)$ really is, and equivalences and natural transformations - this will allow us to compare module categories over different rings.