

## Exercises on Hom-Ext sequences

(1) Let  $0 \rightarrow X \rightarrow Y \rightarrow Z \rightarrow 0$  be a short exact sequence of finite dimensional modules. Assume that  $Y \cong X \oplus Z$ . Show that the seq splits.

(2) Let  $K$  be a field and  $A = K[x]/x^3$ . Let  $X = K[x]/x^2$  and  $Y = K[x]/x^2$ .

Compute:  $\text{Ext}_A^1(X, X)$

$\text{Ext}_A^1(X, Y)$  and  $\text{Ext}_A^1(Y, X)$

$\text{Ext}_A^1(Y, Y)$

Give in each case exact sequences representing a  $K$ -basis of  $\text{Ext}$ ?

Are there similar non-split exact sequences for modules over  $K[x]/x^n$ ?

What about finite dimensional  $K[x]$ -modules?