Exercises on quiver representations

(1) Let $Q = \overset{1}{\bullet} \underset{\beta}{\overset{\alpha}{\rightrightarrows}} \overset{2}{\bullet}$ be the Kronecker quiver, $K = \overline{K}$ an algebraically closed field and $A = KQ$ the path algebra.

(a) Write A_A as a direct sum of two quiver representations, $P(1)$ and $P(2)$.

(b) Let $S(1)$ and $S(2)$ be non-isomorphic simple representations, associated with vertices. Determine all homomorphisms between these representations and $P(1)$ and $P(2)$, in either direction.

(c) Compute $\mathrm{Ext}_A^1(S(1), S(2))$ and determine the middle terms in each equivalence class of extensions. How many pairwise non-isomorphic modules occur as middle terms altogether?

(d) For each E you found in (c) as a middle term determine all self-extensions $\mathrm{Ext}_A^1(E,E)$ and the middle terms occuring there. Try to write the matrices $E'(\alpha)$ and $E'(\beta)$ in a "normal" form for each E' occurring as middle term.

(2) Let KQ be finite dimensional (for some Q). Let $A = KQ$, then $D(_A A)$ is an injective right A-module. Decompose $D(_A A)$ into a direct sum of representations, one summand for each vertex in Q_0 and determine a basis of $D(_A A)$ corresponding (in a way to be made precise) to paths ending in vertex i when considering the summand of $D(_A A)$ associated with $i \in Q_0$.

(3) Let $Q = \bullet \circlearrowleft^{\rho}$ and $K = \overline{K}$. Let S and T be simple representations, $S \not\cong T$. Compute $\mathrm{Ext}^1_{KQ}(S,T)$ and $\mathrm{Ext}^1_{KQ}(S,S)$ and find "normal ~~forms~~ forms" for the middle terms occuring. Compute self-extensions of these middle terms.