More exercises on Ext^1

(1) Let $A = k[x,y]$ for k a field, $X = A/\langle y\rangle$ and $Y = A/\langle x\rangle$. Compute $\mathrm{Ext}^1_A(X,Y)$ and describe the extensions.

(2) Let $A = \begin{pmatrix} k & k & k \\ 0 & k & k \\ 0 & 0 & k \end{pmatrix}$ for k a field, $P_1 = \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}$, $P_2 = \begin{pmatrix} k \\ k \\ 0 \end{pmatrix}$ and $P_3 = \begin{pmatrix} k \\ k \\ k \end{pmatrix}$ projective left modules, $S_1 = P_1$, $S_2 = P_2/S_1$, $S_3 = P_3/P_2$ simple modules and $M = P_3/S_1$. Compute $\mathrm{Ext}^1_A(M, S_1)$, $\mathrm{Ext}^1_A(M, S_2)$, $\mathrm{Ext}^1_A(M, P_2)$ and $\mathrm{Ext}^1_A(M, P_3)$ and describe the extensions.

(3) Let $0 \to X \to Y \to Z \to 0$ be an extension of A-modules and $\alpha: X \to X'$ ~~and~~ and $\beta: Z' \to Z$ A-module homomorphisms. Show that the following diagrams are ~~exact~~ commutative with exact rows.

$$\begin{array}{ccccccccc}
0 & \to & X & \to & \text{pullback} & \to & Z' & \to & 0 \\
 & & \| & & \downarrow & & \downarrow \beta & & \\
0 & \to & X & \to & Y & \to & Z & \to & 0
\end{array}$$

and

$$\begin{array}{ccccccccc}
0 & \to & X & \to & Y & \to & Z & \to & 0 \\
 & & \alpha\downarrow & & \downarrow & & \| & & \\
0 & \to & X' & \to & \text{pushout} & \to & Z & \to & 0
\end{array}$$

So, there are induced maps $\mathrm{Ext}^1_A(Z, X) \to \mathrm{Ext}^1_A(Z', X)$ and $\mathrm{Ext}^1_A(Z, X) \to \mathrm{Ext}^1_A(Z, X')$. Determine these maps in the situation of (2) for $\beta: S_2 \to M (= Z)$ and X as in (2), and for $Z = M$, $\alpha: S_1 \to P_2$ and $\alpha: P_2 \to P_3$.