Exercises on computing Ext^1 by using Theorem 3.3

(1) Compute all extensions of $\mathbb{Z}/2\mathbb{Z}$ by $\mathbb{Z}$. Give representatives.
The same question for $\mathbb{Z}/3\mathbb{Z}$ instead of $\mathbb{Z}/2\mathbb{Z}$.

(2) Let $A = \begin{pmatrix} \mathbb{C} & \mathbb{C} \\ 0 & \mathbb{C} \end{pmatrix}$, over a field $\mathbb{C}$, $e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ and $e_2 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$.
Denote by P_1 and P_2 the projective modules

$$P_1 = Ae_1 \text{ and } P_2 = Ae_2$$

and by S_1 and S_2 the simple modules $S_1 = P_1$ and $S_2 = P_2/\alpha(S_1)$, where $\alpha : S_1 \to P_2$ is a non-zero homomorphism.
Compute $\mathrm{Ext}^1_A(X,Y)$ for $X, Y \in \{P_1, P_2, S_2\}$ and give non-split extensions whenever there can be one.
Is S_2 projective? Which modules can be injective?

(3) Notation and setup as in (2). Determine $D(A_A)$ and thus decide which of the modules in (2) are injective.