Exercises on t-structures

(1) Let k be a field. Determine all t-structures on $D^b(k\text{-mod})$.

(2) Let $\mathcal{T}$ be a triangulated category and $\mathcal{U}$ a full additive subcategory. The perpendicular category $\mathcal{U}^\perp$ ("$\mathcal{U}$ perp") is the full additive subcategory of $\mathcal{T}$ whose objects Y satisfy $\mathrm{Hom}_{\mathcal{T}}(X,Y)=0$ for all X in $\mathcal{U}$. We consider the following conditions:

(a) $\mathcal{U}[1] \subset \mathcal{U}$

(b) For each distinguished triangle $X \to Y \to Z \rightsquigarrow X[1]$ in $\mathcal{T}$, Y is in $\mathcal{U}$ provided X and Z are in $\mathcal{U}$.

(c) The inclusion $\mathcal{U} \to \mathcal{T}$ admits a right adjoint $\mathcal{T} \to \mathcal{U}$, $X \mapsto X_{\mathcal{U}}$.

(d) For each X in $\mathcal{T}$ there exists a distinguished triangle $X_{\mathcal{U}} \to X \to X_{\mathcal{U}^\perp} \rightsquigarrow X_{\mathcal{U}}[1]$ with $X_{\mathcal{U}}$ in $\mathcal{U}$ and $X_{\mathcal{U}^\perp}$ in $\mathcal{U}^\perp$.

Show that (a), (b) and (c) is equivalent to (a) and (d).

Show that $\mathcal{U} \mapsto (\mathcal{U}, \mathcal{U}^\perp[1])$ gives a bijection between $\mathcal{U}$ satisfying the above conditions and the t-structures on $\mathcal{T}$.

(3) Let $A = kQ$ for $Q = \circ \longrightarrow \circ$, P_1 simple projective, P_2 projective and injective.

Let $T_1 = \begin{matrix} 0 \to P_1 \to 0 \\ \oplus \\ 0 \to P_1 \to P_2 \to 0 \end{matrix}$ and $T_2 = \begin{matrix} 0 \to P_1 \to 0 \\ \oplus \\ 0 \to P_2 \to 0 \end{matrix}$

Check for each complex, which properties of a tilting complex it satisfies. If it is a tilting complex, determine the corresponding derived equivalence.