

## Exercises on computing extensions in homotopy or derived categories

(1) Let  $R$  be a ring and  $X, Y \in R\text{-Mod}$ . Is  $\text{Ext}_R^n(X, Y) \cong \text{Hom}_{D(A)}(X, Y[n])$  for all  $n \in \mathbb{Z}$ ?

↓ (throughout: try to compute with complexes)

(2) Let  $A = k[x]/(x^2)$ . Compute  $\text{Hom}_{D(A)}(X, Y[n])$  for all  $n \in \mathbb{Z}$  for the following objects  $X, Y$ :  ${}_A A, k$  (simple  $A$ -module),  $0 \rightarrow A \xrightarrow{f} A \xrightarrow{f} \dots \xrightarrow{f} A \xrightarrow{f} A \rightarrow 0$  where  $f: A \rightarrow A$  sends  $1$  to  $x$ ,  $x$  to  $0$ .

(3) Let  $A = kQ$ , the path algebra of the quiver  $0 \rightarrow 0 \rightarrow 0$ .

(a) Determine all  $\text{Ext}_A^n(X, X)$  for  $n \in \mathbb{Z}$ ,  $X$  indecomposable in  $D(A)$ .  
 $\text{Hom}_{D(A)}(X, X[n])$

(b) For which  $n \in \mathbb{Z}$  can  $\text{Hom}_{D(A)}(X, Y[n])$  be non-zero for some indecomposable objects  $X, Y$  in  $D(A)$ ?

(c) Determine all modules  ${}_A X = X_1 \oplus X_2 \oplus X_3$  with  $X_1, X_2, X_3$  indecomposable, pairwise non-isomorphic such that  $\text{Ext}_A^n(X, X) = 0 \forall n \neq 0$ .

For each such  $X$ , compute the endomorphism ring, and  $\text{pdim}(X)$ .

For each such  $X$ , decide if in  $D(A)$  the regular module  ${}_A A$  is isomorphic to a finite complex  $0 \rightarrow M_1 \rightarrow M_2 \rightarrow \dots \rightarrow M_n \rightarrow 0$  (some  $n$ ) with all  $M_i \in \text{add } X$ .

(4) Let  $A$  be a finite dimensional algebra,  $e \in A$  ( $e = e^2$ ) indecomposable projective and  $T = 0 \rightarrow 0 \rightarrow A(1-e) \rightarrow 0$

$$\begin{array}{ccccccc} & & \oplus & & & & \\ & & & & & & \text{for some } P, Q \text{ projective} \\ 0 & \rightarrow & P & \xrightarrow{f} & Q & \rightarrow & 0 \end{array}$$

Give criteria on  $P, Q$  and  $f$  such that  $\text{Hom}_{D(A)}(T, T[n]) = 0 \forall n \neq 0$ .

Give examples of such complexes  $T$ , where  $P \neq 0$ , for explicit algebras, eg for path algebras of quivers. Compute then the endomorphism ring of  $T$  in  $D(A)$ .