Preview

When R and S are rings we can compare them by checking if they are isomorphic. We also can assign to R and S the module categories R-Mod and S-Mod and compare these by checking if they are equivalent. Morita equivalence of rings is much more general than isomorphism of rings.
In a next step we can assign to R and S their derived module categories. Derived equivalence is much more general than Morita equivalence.

$$R \qquad S$$

$$R \simeq S$$

$$\Downarrow$$

$$R\text{-Mod} \simeq S\text{-Mod}$$

$$\Downarrow$$

$$D(R\text{-Mod}) \simeq D(S\text{-Mod})$$

In this chapter we first construct the derived category $D(R\text{-Mod})$ from the homotopy category $K(R\text{-Mod})$ by a technique called calculus of fractions. In this way, quasi-isomorphisms become invertible morphisms, and in the derived module category $D(R\text{-Mod})$ a module M isomorphic to each of its projective or injective resolutions.

The homotopy category $K(R\text{-Mod})$ and the derived category $D(R\text{-Mod})$ are additive categories, but not abelian (in generell). They have a different structure: they are triangulated categories. We will define this structure and verify it for both categories. The derived equivalences to be studied later on will, by definition, preserve this triangulated structure.