More exercises on triangulated categories

(1) As before, $\tilde{A}_n$ is the cyclic quiver  with n vertices, $n \in \mathbb{N}$, k a field.
Let $A := k\tilde{A}_n/_I$ be a finite dimensional quotient algebra satisfying $\mathrm{rad}^2(A) = 0$.

(a) Which algebras A of this form are self-injective?

From now on, let A be self-injective.

(b) Determine the objects and morphisms in A-$\underline{\mathrm{mod}}$ and the triangulated structure, including the shift.

(c) Determine all auto-equivalences $F: A\text{-}\underline{\mathrm{mod}} \xrightarrow{\sim} A\text{-}\underline{\mathrm{mod}}$ of the additive category A-$\underline{\mathrm{mod}}$.

(d) Determine all auto-equivalences $G: A\text{-}\underline{\mathrm{mod}} \xrightarrow{\sim} A\text{-}\underline{\mathrm{mod}}$ of the triangulated category A-$\underline{\mathrm{mod}}$.

(e) Let $A = k\tilde{A}_n/_I$, $B = k\tilde{A}_{n_1}/_{I_1}$ and $C = k\tilde{A}_{n_2}/_{I_{n_2}}$. Suppose the categories A-$\underline{\mathrm{mod}}$ and $(B \oplus C)$-$\underline{\mathrm{mod}}$ are equivalent, and $\mathrm{rad}\, A \neq 0$, $\mathrm{rad}\, B \neq 0$, $\mathrm{rad}\, C \neq 0$.
Does it follow that $A \simeq B \oplus C$? Or at least $n = n_1 + n_2$?
Does it help to assume A is indecomposable?

(f) Same questions as in (e), but now A-$\underline{\mathrm{mod}} \simeq (B \oplus C)$-$\underline{\mathrm{mod}}$ as triangulated categories.

(2) Let $\mathcal{A} = \mathbb{Z}$-mod, $Ch(\mathcal{A})$ the category of complexes and $D(\mathcal{A})$ the derived category.

(a) Are there morphisms f in $Ch(\mathcal{A})$ such that f is not nullhomotopic, but 0 in $D(\mathcal{A})$?

(b) Are there morphisms g in $Ch(\mathcal{A})$ such that $g \neq 0$ in $D(\mathcal{A})$, but g induces 0 on cohomology?