

## Gabriel quivers and Auslander-Reiten quivers

Over an algebraically closed field  $K$ , a finite dimensional algebra is Morita equivalent to a bound quiver algebra  $KQ/I = A$ .  $Q$  is uniquely defined by  $A/\text{rad} A$  and  $\text{rad} A/\text{rad}^2 A$ . Let us call  $Q$  the Gabriel quiver of  $A$ .

The following exercises are supposed to exhibit an analogy between Gabriel quivers and Auslander-Reiten quivers. In particular, the Auslander-Reiten quiver of an algebra of finite representation type "is" the Gabriel quiver of another finite dimensional algebra.

(a) Let  $A$  be an algebra and  $M_A$  an  $A$ -module. Set  $E := \text{End}_A(M)$ . Use Yoneda's Lemma to prove that  $\text{Hom}_A(M, -)$  induces an equivalence

$$\begin{array}{ccc} \text{add } M & \xrightarrow{\sim} & \text{proj } E \\ \uparrow & & \uparrow \\ \text{direct summands of } & & \text{finitely generated} \\ M^n, n \in \mathbb{N}_0 & & \text{projective modules} \end{array}$$

(You may also show that  $D \text{Hom}_A(-, T)$  yields the injective  $E$ -modules.)

Why is this called projectivisation?

(b) Let  $A$  have finite representation type and let  $M := M_1 \oplus \dots \oplus M_n$  be the direct sum of a complete set of representatives of isomorphism classes of indecomposable  $A$ -modules. Let  $E := \text{End}_A(M)$  as before. ( $E$  is called the Auslander algebra of  $A$ .)

Compare the AR quiver of  $A$  with the Gabriel quiver of  $E$ : (number of) vertices, (number of) arrows, elements of  $\text{rad}$ ,  $\text{rad}^2$ , ~

(c) Compute the AR quiver of  $A$  and the Gabriel quiver of  $E$  (as in (b)) for

$$A_1 = K[x]/(x^2), \quad A_2 = K[x]/(x^3), \quad A_3 = K(\cdot \rightarrow \cdot)$$