Exercises

(1) Derive Auslander's defect formula from the Auslander–Reiten formulae.

(2) Compute $D\,Tr$ for all indecomposable modules in the following situations:

(a) $A_1 = k(\to\to)$ (6 indecomposable modules)

(b) $A_2 = k(\to\leftarrow)$ (6 indecomposable modules)

(c) $A_3 = k[x]/(x^3)$ (3 indecomposable modules)

(3) Let A be a finite dimensional algebra and $P(A)$ the additive category of finite dimensional projective A-modules. The morphism category $Mor(P(A))$ has objects: $P \xrightarrow{f} Q$ ($P, Q \in P(A)$, f a module homomorphism) and morphisms:

$$\begin{tikzcd} P_1 \arrow[r, "f"] \arrow[d, "\alpha"'] \arrow[dr, phantom, "\circlearrowleft"] & Q_1 \arrow[d, "\beta"] \\ P_2 \arrow[r, "g"] & Q_2 \end{tikzcd}$$

(module homomorphisms α, β such that the square commutes)

Define a functor $F: Mor(P(A)) \longrightarrow A\text{-mod}$ which is dense and full, but not in general faithful

Modify $Mor(P(A))$ such that F induces an equivalence G between the modified category and A-mod.