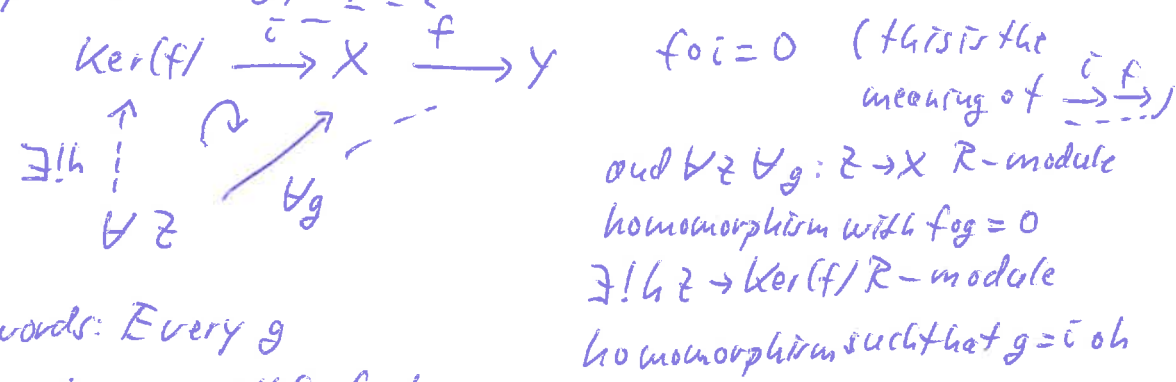


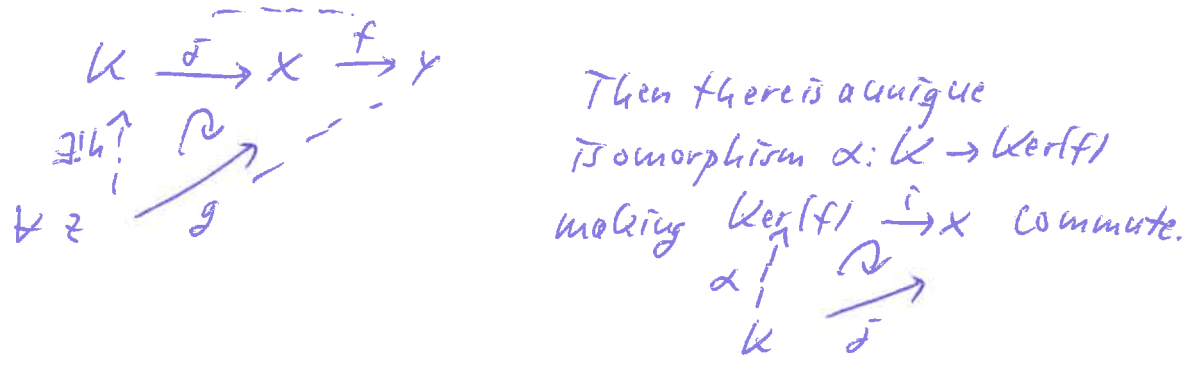
Practising universal properties

(1) Let $f: X \rightarrow Y$ be an R -module homomorphism. Show that $\text{Ker}(f)$ and the inclusion $i: \text{Ker}(f) \rightarrow X$ satisfy the following property:

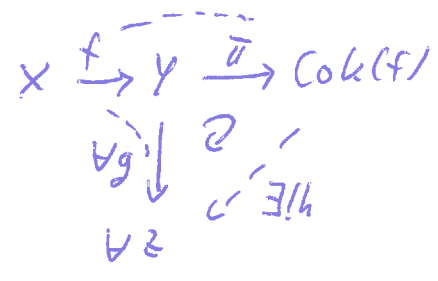


(Another words: Every g precomposing to zero with f factors uniquely through i .)

Suppose $K \xrightarrow{j} X$ has the same property:



(2) Define the cokernel $\text{Cok}(f)$ dually:



Show that $\text{Cok}(f)$ exists and is unique up to unique isomorphism.

Note that $\text{Ker}(f)$ is a pair: $(\text{Ker}(f), i)$, an object with a morphism.

And $\text{Cok}(f)$ is a pair $(\text{Cok}(f), \bar{u})$.

Thus one can define $\text{Cok}(\text{Ker}(f)) := \text{Cok}(i)$ and $\text{Ker}(\text{Cok}(f)) := \text{Ker}(\bar{u})$.

(3) Show that $\text{Ker}(\text{Cok}(f)) \cong \text{Cok}(\text{Ker}(f))$ and both are isomorphic to $\text{Im}(f)$ (which in this way can be described abstractly!).

Hint:

$$\begin{array}{ccccc} \text{Ker}(f) & \xrightarrow{\bar{c}} & X & \xrightarrow{f} & Y & \xrightarrow{\bar{a}} & \text{Cok}(f) \\ & & \downarrow & \nearrow \exists & \uparrow & & \\ & & \text{Cok}(f) & \xrightarrow{\exists!} & \text{Ker}(\bar{a}) & & \end{array}$$

(4) What is the problem when we try to do the same for groups instead of \mathcal{R} -modules?