

Triangulated categories - exercises

- (1) Let \mathcal{T} be a triangulated category and E a full triangulated subcategory of \mathcal{T} . Show that the following two conditions are equivalent to each other:
- If $f : X \rightarrow Y$ is contained in a triangle $X \rightarrow Y \rightarrow Z \rightsquigarrow$ with Z an object in E and if f factors through an object in E , then X and Y are objects in E .
 - Objects of \mathcal{T} that are direct summands of objects of E are themselves objects of E .
- (2) Let $A = k$ be a field.
- Determine the bounded derived category $D^b(A - \text{mod})$.
 - Determine the stable category $\hat{A} - \underline{\text{mod}}$ of the repetitive algebra \hat{A} . Compare the two triangulated categories. Is any of these categories abelian? Let $B = k[x]/x^2$ and address the same questions.
- (3) Let A be a ring.
- Let X be an object in $D^b(A - \text{Mod})$. Show that X is isomorphic to a bounded complex of projective A -modules if and only if for all objects Y in $D^b(A - \text{Mod})$ the morphism space $\text{Hom}_{D^b(A - \text{Mod})}(X, Y[n])$ vanishes for large n . Let B be another ring. Show that any equivalence of triangulated categories

$$D^b(A - \text{Mod}) \simeq D^b(B - \text{Mod})$$
 restricts to an equivalence $K^b(A - \text{Proj}) \simeq K^b(B - \text{Proj})$.
- (4) Let Q be the quiver $1 \rightarrow 2 \rightarrow 3$ and k a field. Determine the derived module category $D^b(kQ - \text{mod})$. Let B be the quotient of kQ modulo the path of length two. Determine the derived module category $D^b(B - \text{mod})$. Compare the two derived categories.

Homepage of the course:

<http://www.iaz.uni-stuttgart.de/LstAGeoAlg/Koenig/TriangCat/TriangCat.t>