Problem sheet 2 Thursday, 23rd of November, 2017

**Triangulated categories - exercises**

(1) Let $\mathcal{T}$ be an additive category with a set of distinguished triangles satisfying (TR1), (TR2) and (TR3), and let $X \xrightarrow{f} Y \xrightarrow{g} Z \xrightarrow{h}$ and $U \xrightarrow{r} V \xrightarrow{s} W \xrightarrow{t}$ be candidate triangles. Show that the following two statements are equivalent:
(a) Both candidate triangles are distinguished.
(b) The triangle $X \oplus U \xrightarrow{(f,r)} Y \oplus V \xrightarrow{(g,s)} Z \oplus W \xrightarrow{(h,t)}$ is distinguished.

(2) Let $\mathcal{T}$ be an additive category with a set of distinguished triangles satisfying (TR1), (TR2) and (TR3), and let $X \xrightarrow{f} Y$ be a morphism. Show that the following statements are equivalent:
(a) The morphism $f$ has a kernel.
(b) The morphism $f$ has a cokernel.
(c) The morphism $f$ is isomorphic to a map $U \oplus Z \to Z \oplus V$ that is the identity on $Z$ and zero on $U$, for some $U$, $V$ and $Z$.

(3) Let $\mathcal{T}$ and $\mathcal{T}'$ be additive categories with sets of distinguished triangles satisfying (TR1), (TR2) and (TR3), and let $F : \mathcal{T} \to \mathcal{T}'$ be an exact (or: triangulated) functor, that is, $F$ sends distinguished triangles to distinguished triangles and $F$ commutes with the shift (up to natural isomorphism). Show that $F$ is an additive functor.

(4) Let $A$ be a Nakayama algebra with $rad(A)^2 = 0$. Determine all auto-equivalences and all triangulated auto-equivalences of $A - \underline{mod}$.

(5) Let $A$ be a finite dimensional $k$-algebra. Define $T(A)$ to be the vector space $A \oplus D(A)$, where $D$ is $k$-duality, and define a multiplication on $T(A)$ such that $D(A)$ is a two-sided ideal which multiplies with itself to zero. Show that $T(A)$ is a self-injective algebra.
Define $\hat{A}$ to be an infinite-dimensional algebra (without unit), which in terms of infinite square matrices (with finitely many non-zero entries only) looks as follows:

$$\begin{pmatrix} \ddots & \ddots & & & \\ & A & D(A) & & \\ & & A & D(A) & \\ & & & \ddots & \ddots \end{pmatrix}$$

Show that $\hat{A}$ is a self-injective algebra.
Compare the $\mathbb{Z}$-graded $T(A)$-modules with the $\hat{A}$-modules.

Homepage of the course:
`http://www.iaz.uni-stuttgart.de/LstAGeoAlg/Koenig/TriangCat/TriangCat.t`