Problem sheet 1 Thursday, 2nd of November, 2017

Triangulated categories - exercises

(1) Let A be a finite dimensional algebra of finite global dimension n. Is the global dimension of $\underline{fun}(A-mod)$ finite?

(2) (a) Let A be a finite dimensional algebra. Characterise the property of all indecomposable non-simple projective A-modules being injective, in terms of the category $\underline{fun}(A-mod)$.
(b) Suppose A is stably equivalent to a self-injective algebra B. Are there projective A-modules that must be injective?

(3) Let A be a finite dimensional algebra, J its Jacobson radical and suppose $J^2 = 0$. Let $H := \begin{pmatrix} A/J & J \\ 0 & A/J \end{pmatrix}$. Show that H is a hereditary algebra. Define a functor

$$F : A - mod \to H - mod$$

which induces a functor G between the stable categories. Show that G is an equivalence when $A = k[x]/x^2$. Find more examples when G is an equivalence.

(4) (a) Let A and B be finite dimensional algebras over k and let ${}_AM_B$ and ${}_BN_A$ be bimodules that are projective on either side. Suppose that $M \otimes_B N \simeq A \oplus P$ as a bimodule, where P is projective as a bimodule (that is, P is a projective $A \otimes_k A^{op}$-module), and that $N \otimes_A M \simeq B \oplus Q$, where Q is projective as a bimodule. Show that A and B are stably equivalent.
(b) Let A be self-injective. Is the stable auto-equivalence of A induced by the Heller operator Ω an example of (a)?

(5) Let A be a finite dimensional algebra and B a subalgebra (with the same unit). Give conditions on A and B ensuring that induction and restriction induce stable equivalences, and give non-trivial examples where the conditions are satisfied.

Homepage of the course:
`http://www.iaz.uni-stuttgart.de/LstAGeoAlg/Koenig/TriangCat/TriangCat.t`