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Sheet 11

Problem 25 Let $\mathcal{Z}$ be a grading category.
Let $A = (A, (m_1), (A^{\langle i \rangle})_i)$ be a minimal eA_1-algebra over $\mathcal{Z}$.
Suppose that there exist shift-graded linear map $d^{\langle i \rangle} : A^{\langle i \rangle} \to A^{\langle i-1 \rangle}$ of degree 1 and shift-graded linear map $e^{\langle i \rangle} : A^{\langle i \rangle} \to A^{\leqslant i-2}$ of degree 1 for $i \in \mathbf{Z}_{\geqslant 0}$ such that

$$\iota^{\langle i \rangle} \cdot m_1 \;=\; d^{\langle i \rangle} \cdot \iota^{\langle i-1 \rangle} + e^{\langle i \rangle} \cdot \iota^{\leqslant i-2} \,.$$

holds for $i \in \mathbf{Z}_{\geqslant 0}$.

(1) Express the Stasheff equation at 1 in terms of $d^{\langle i \rangle}$ and $e^{\langle i \rangle}$, where $i \in \mathbf{Z}_{\geqslant 0}$.

(2) Show that A is diagonally resolving if and only if $\mathrm{Kern}(d^{\langle i \rangle}) = \mathrm{Im}(d^{\langle i+1 \rangle})$ for $i \in \mathbf{Z}_{\geqslant 1}$.

Problem 26 Let $\mathcal{Z}$ be a grading category.
Suppose given an eA_∞-algebra $(A, (m_k)_k \,, (A^{\langle i \rangle})_i)$ over $\mathcal{Z}$. Suppose that $A^{\langle i \rangle} = 0$ for $i \in \mathbf{Z} \smallsetminus [0, \ell]$.
For which $k \in \mathbf{Z}_{\geqslant 1}$ is the Schmid condition on m_k not void?
For which $k \in \mathbf{Z}_{\geqslant 1}$ is the strong Schmid condition on m_k not void?

(1) Consider the case $\ell = 1$.

(2) Consider the case $\ell = 2$.

(3) Consider the case $\ell = 3$.

Problem 27 Let $\mathcal{Z}$ be a grading category.
Suppose given an eA_∞-algebra $(A, (m_k)_k, (A^{\langle i \rangle})_i)$ over $\mathcal{Z}$. Let $k \geqslant 1$. Let $(j_1, \ldots, j_k) \in \mathbf{Z}_{\geqslant 0}^{\times k}$.
What bound results from the Schmid condition for the image of $A^{\langle j_1 \rangle} \otimes \ldots \otimes A^{\langle j_k \rangle}$ under a summand of the Stasheff equation at k ?

Problem 28 Let $X = (X, \leqslant)$ be a poset. We call X *artinian* if it does not contain a strictly descending chain. We call X *superartinian* if $X_{\leqslant \xi}$ is finite for all ξ. We call X *discrete* if $(\leqslant) = (=)$. We call X *narrow* if each discrete subposet of X is finite.
Suppose given $k \in \mathbf{Z}_{\geqslant 1}$ and posets $Y_1 \,, \ldots, \, Y_k$.

(1) Show that X is artinian if and only if each nonempty subposet of X has a minimal element.

(2) If X is superartinian, show that X is artinian. Does the converse hold?

(3) Construct the product $\prod_{i \in [1,k]} Y_i$ in Poset, which is to be equipped with monotone maps $\prod_{i \in [1,k]} Y_i \xrightarrow{\pi_j} Y_j$ for $j \in [1,k]$ such that for each poset T and each tuple $(T \xrightarrow{t_i} Y_i)_i$ of monotone maps, there exists a unique monotone map $T \xrightarrow{t} \prod_{i \in [1,k]} Y_i$ such that $t \cdot \pi_j = t_j$ for $j \in [1,k]$.

(4) If Y_i is artinian for $i \in [1,k]$, show that $\prod_{i \in [1,k]} Y_i$ is artinian.

(5) If Y_i is superartinian for $i \in [1,k]$, show that $\prod_{i \in [1,k]} Y_i$ is superartinian.

(6) Show that $\mathbf{Z}_{\geqslant 0}^{\times k} := \prod_{i \in [1,k]} \mathbf{Z}_{\geqslant 0}$ is superartinian and narrow.