Spectral Theory and Dynamics of Quantum Systems

GRADUIERTENKOLLEG 1838

Guest Lectures

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Estimates on Functional Integrals of Quantum Mechanics and Non-Relativistic Quantum Field Theory

November 3, 2016 - 10:00 h,
Room 8.526 - University of Stuttgart, Pfaffenwaldring 57

Abstract: We provide a unified method for obtaining upper bounds for certain functional integrals appearing in quantum mechanics and non-relativistic quantum field theory, functionals of the form $E\left[\exp(A_T)\right]$, the (effective) action A_T being a function of particle trajectories up to time T. The estimates in turn yield rigorous lower bounds for ground state energies, via the Feynman-Kac formula. The upper bounds are obtained by writing the action for these functional integrals in terms of stochastic integrals. The method is first illustrated in familiar quantum mechanical settings: for the hydrogen atom, for a Schr\"odinger operator with $1/|x|^2$ potential with small coupling, and, with a modest adaptation of the method, for the harmonic oscillator. We then present our principal applications of the method, in the settings of non-relativistic quantum field theories for particles moving in a quantized Bose field, including the optical polaron and Nelson models. – Joined work with L. E. Thomas

A Lower Bound on the Renormalized Nelson Model

November 10, 2016 - 10:00 h,
Room 8.526 - University of Stuttgart, Pfaffenwaldring 57

Abstract: We provide explicit lower bounds for the ground-state energy of the renormalized Nelson model in terms of the coupling constant α and the number of particles N, uniform in the meson mass and valid even in the massless case. In particular, for any number of particles N and large enough α we provide a bound of the form $-Ca^2N^3log^2(aN)$, where C is an explicit positive numerical constant; and if α is sufficiently small, we give one of the form $-Ca^2N^3log^2N$ for $N\geq2$, and $-Ca^2$ for $N=1$. Whereas it is known that the renormalized Hamiltonian of the Nelson model is bounded below (as realized by E. Nelson) and implicit lower bounds have been given elsewhere (as in a recent work by Gubinelli, Hiroshima, and Lörinczi), ours seem to be the first fully explicit lower bounds with a reasonable dependence on α and N. We emphasize that the logarithmic term in the bounds above is probably an artifact in our calculations, since one would expect that the ground-state energy should behave as $-Ca^2N^3$ for large N or α, as in the polaron model of H. Fröhlich.