M. Künzer

$A_\infty$-categories, WS 16/17

# Sheet 13

**Problem 30** Suppose given a grading category $\mathcal{Z}$. Suppose given $n \in \mathbf{Z}_{\geqslant 0}$.
Suppose given a piecewise projective $\mathcal{Z}$-graded module $M_i$ for $i \in [1, n]$.
Show that $\bigotimes_{i\in[1,n]} M_i$ is piecewise projective.

**Problem 31** Suppose we are in the setup of §2.4.
In particular, we consider the poset $\mathfrak{A}$ of admissible triples.
A subposet $Y$ of a poset $X$ is called a *lower* subposet if for $y \in Y$ and $x \in X$ with $x \leqslant y$, we may conclude that $x \in Y$.

(1) Suppose given a poset $X$ and a subposet $Y \subseteq X$. Show that $Y \cap \min(X) \subseteq \min(Y)$.
If $Y$ is a lower subposet in $X$, show that $Y \cap \min(X) = \min(Y)$.

(2) Show that for each totally ordered subposet $T$ of $\mathfrak{A}$, there exists a unique element $s \in \mathfrak{A}$ such that $t \leqslant s$ for $t \in T$ and such that whenever given $s' \in \mathfrak{A}$ such that $t \leqslant s'$ for $t \in T$, then $s \leqslant s'$. Write $s =: \sup T$.

(3) Using the Lemma of Zorn and Lemma 64, show that for each element $x = (L, M, Q) \in \mathfrak{A}$ there exists an element $x' = (L', M', Q') \in \mathfrak{A}$ such that $x \leqslant x'$ and such that $L' = \mathbf{Z}_{\geqslant 0}^{\times n}$.

(4) Show the assertion of (3) again. Use Lemma 64 to do so. But do not use the Lemma of Zorn. Rather, form an ascending chain $x = x_0 \leqslant x_1 \leqslant \ldots$ such that, writing $x_k = (L_k, M_k, Q_k)$ for $k \in \mathbf{Z}_{\geqslant 0}$, we have $L_{k+1} = L_k \cup \min(\mathbf{Z}_{\geqslant 0} \smallsetminus L_k)$. Then take $x' = \sup\{\, x_k \,:\, k \in \mathbf{Z}_{\geqslant 0} \,\}$.