Convergence of a staggered Lax-Friedrichs scheme on unstructured 2D-grids

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Abstract. Based on Nessyahu's and Tadmor's nonoscillatory central difference schemes for one-dimensional hyperbolic conservation laws [14], for higher dimensions, several finite volume extensions and numerical results on structured and unstructured grids have been presented. The experiments show the wide applicability of these multidimensional schemes.

The theoretical arguments which support this, are some maximum-principles and a convergence proof in the scalar linear case. A general proof of convergence, as obtained for the original one-dimensional NT-schemes, does not exist for any of the extensions to multidimensional nonlinear problems.

For the finite volume extension on two-dimensional unstructured grids introduced by Arminjon and Viallon [3] we can prove convergence for the first order scheme in the case of a nonlinear scalar hyperbolic conservation law.

1. Introduction

The Nessyahu-Tadmor schemes, introduced in [14], are Godunov-type schemes for hyperbolic conservation laws. Their characteristic property is the use of two alternating staggered grids combined with MUSCL-type linear reconstruction and a predictor step, which yield second order accuracy. Due to the staggering the need of solving local generalized Riemann-problems at cell interfaces is bypassed. Easy application to systems of hyperbolic conservation laws is possible due to this. For these one-dimensional schemes theoretical foundation was established by the proof of convergence to the unique entropy solution for the scalar genuinely nonlinear case in the introductory paper of Nessyahu and Tadmor. Modifications of the schemes which avoid staggered grids have been proposed in [9], [13].

The idea of the construction was extended to two-dimensional cartesian grids by Arminjon, Stanescu, Viallon [2]. A discrete maximum-principle for a slightly different extension was obtained by Jiang and Tadmor in the scalar case [10].

A formulation of the NT-schemes by Arminjon, Viallon for two-dimensional unstructured grids was presented, cf. [3], and convergence has been proven for the case of a linear hyperbolic equation [4]. Recently an extension of the scheme to three space dimensions has been proposed by Arminjon, Madrane and St-Cyr [1].

We consider the Cauchy-problem given by the fully nonlinear scalar conservation law with flux $\vec{f} = (f_1, f_2)^T$ and initial values u_0

$$u_t + \nabla \cdot \vec{f}(u) = 0 \quad in \quad \mathbb{R}^2 \times [0,T], \tag{1}$$

$$u(\cdot,0) = u_0 \quad in \quad \mathbb{R}^2. \tag{2}$$

Our result is summed up in Theorem 4.1. We consider the basic first order scheme as proposed by Arminjon and Viallon for unstructured grids, which is the most simple NT-scheme, the staggered Lax-Friedrichs scheme.

We can show that any sequence of discrete solutions defined by this staggered Lax-Friedrichs scheme converges to the unique entropy solution of the Cauchy-problem. For this we need regularity of the data, i.e. $\vec{f} \in C^1(\mathbb{R})^2$, $u_0 \in L^1(\mathbb{R}^2) \cap L^\infty(\mathbb{R}^2)$, and a non-degeneracy condition on the underlying sequence of refining space-time-grids. This is formulated by a CFL and inverse CFL-condition for the time-discretization Δt and by geometrical bounds for the finite volume cells.

To obtain this result we make use of measure-valued solutions, particularly means of DiPerna [7], which have been applied in several other convergence proofs, e.g. [11], [6], [12]. After collecting some of these results concerning measure-valued solutions in a general convergence theorem, Thm. 2.2, it remains to show several properties of the sequence of numerical solutions. These properties will be given in Section 4. Proofs however can not be given in this limited proceedings contribution, for these we refer to our forthcoming paper [8].

2. Convergence Framework

Notation 2.1. *Let $Prob(K)$ denote the space of probability-measures on the compact set $K \subset \mathbb{R}$. For all $g \in C^0(\mathbb{R}), \mu \in Prob(K)$ we denote*

$$\langle \mu, g \rangle := \int_K g d\mu.$$

Definition 2.2 (Young-measure, emv-solution)**.** *A (uniformly bounded)* **Young-measure** *is a map $\nu : \mathbb{R}^2 \times [0,T] \to Prob(K)$ for some compact $K \subset \mathbb{R}$ such that for all $g \in C^0(\mathbb{R})$ the map $\langle \nu, g \rangle(x,t) := \langle \nu(x,t), g \rangle$ is measurable.*

A Young-measure ν is an **entropy measure-valued solution** *of the conservation law (1) if*

$$\int_0^T \int_{\mathbb{R}^2} \left(\langle \nu, \mathrm{id} \rangle \varphi_t + \langle \nu, \vec{f} \rangle \nabla \varphi \right) = 0,$$

$$\int_0^T \int_{\mathbb{R}^2} \left(\langle \nu, U \rangle \varphi_t + \langle \nu, \vec{F} \rangle \nabla \varphi \right) \geq 0 \tag{3}$$

are satisfied for all entropy pairs $(U, \vec{F})$ (i.e. $U \in C^2(\mathbb{R})$ strictly convex, $\vec{F}' = U' \vec{f}'$) and all $\varphi \in C_0^\infty(\mathbb{R}^2 \times (0,T))$, where in (3) we additionally assume $\varphi \geq 0$. We used the notation $\langle \nu, \vec{f} \rangle := (\langle \nu, f_1 \rangle, \langle \nu, f_2 \rangle)^T$.

This notion of emv-solution naturally extends entropy solutions:

Remark 2.3. *If an entropy measure-valued solution ν is identical to a Dirac-measure δ_u of a function $u(x,t)$ almost everywhere, then the definition exactly states that u is an entropy solution of the conservation law.*

We need several results from the theory of measure-valued solutions in our proof, cf. Tartar [16]. The most important theorem we refer to is due to DiPerna [7]. It states that a Young-measure is identical to a Dirac-measure almost everywhere under certain conditions. In our case this reads as follows.

Theorem 2.1 (DiPerna). *Let $f_1, f_2 \in C^1(\mathbb{R})$, $u_0 \in L^1(\mathbb{R}^2) \cap L^\infty(\mathbb{R}^2)$. Let us further assume the existence of a Young-measure ν such that*

a) *the function $\langle \nu, |\mathrm{id}| \rangle$ is in $L^\infty\left([0,T], L^1(\mathbb{R}^2)\right)$,*
b) *ν is an entropy measure-valued solution to the conservation law (1),*
c) *ν assumes the initial values in the sense that*

$$\lim_{t \searrow 0} \frac{1}{t} \int_0^t \int_{\mathbb{R}^2} \langle \nu_{x,s}, |\mathrm{id} - u_0(x)| \rangle dx ds = 0.$$

Then ν is a.e. identical to the Dirac-measure $\delta_{u(x,t)}$ associated to the unique weak entropy solution $u(x,t)$ of the Cauchy-problem (1), (2), i.e. $\langle \nu_{x,t}, \mathrm{id} \rangle = \langle \delta_{u(x,t)}, \mathrm{id} \rangle = u(x,t)$ almost everywhere

With these tools we obtain a general convergence theorem. This is not a new result, it simply collects sufficient conditions which allow to apply the powerful tools mentioned above. By satisfying these conditions, one can prove convergence of any approximating sequence of functions (not necessarily stemming from a numerical scheme).

Theorem 2.2 (General convergence). *Let $f_1, f_2 \in C^1(\mathbb{R})$, $u_0 \in L^1(\mathbb{R}^2) \cap L^\infty(\mathbb{R}^2)$, $T > 0$, $(u_k)_{k \in \mathbb{N}}$ be a sequence of functions in $L^1_{loc}\left(\mathbb{R}^2 \times [0,T]\right)$, $(h_k)_{k \in \mathbb{N}}$ a sequence of nonnegative real numbers with $\lim_{k \to \infty} h_k = 0$.*
Let nonnegative constants $C_1, C_2, C_\varphi, C_{U,\vec{F},\varphi}$ and $\kappa > 0$ exist, such that for all $k \in \mathbb{N}$ the following conditions hold:

a) $\|u_k\|_{L^\infty(\mathbb{R}^2 \times [0,T])} \le C_1$,
b) $\|u_k(\cdot,t)\|_{L^1(\mathbb{R}^2)} \le C_2$ *for all* $t \in [0,T]$,
c) *for all* $\varphi \in C_0^\infty\left(\mathbb{R}^2 \times [0,T)\right)$

$$\left| \int_0^T \int_{\mathbb{R}^2} \left(u_k \varphi_t + \vec{f}(u_k) \nabla \varphi \right) + \int_{\mathbb{R}^2} u_0 \varphi(\cdot, 0) \right| \le C_\varphi h_k^\kappa,$$

d) *for all* $\varphi \in C_0^\infty\left(\mathbb{R}^2 \times (0,T)\right)$, $\varphi \ge 0$ *and all entropy pairs* $\left(U, \vec{F}\right)$

$$\int_0^T \int_{\mathbb{R}^2} \left(U(u_k) \varphi_t + \vec{F}(u_k) \nabla \varphi \right) \ge -C_{U,\vec{F},\varphi} h_k^\kappa,$$

e) *for* $U(u) \equiv \frac{1}{2}u^2$ *and all* $t \in [0,T]$

$$\int_{I\!R^2} U\left(u_k(\cdot,t)\right) \leq \int_{I\!R^2} U\left(u_0\right).$$

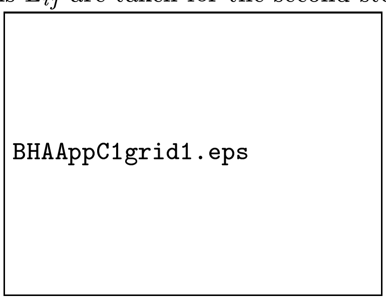
Then the sequence $(u_k)_{k \in I\!N}$ *converges in* $L^q_{loc}\left(I\!R^2 \times [0,T]\right)$ *for all* $1 \leq q < \infty$ *strongly towards the unique entropy solution of the Cauchy-problem (1), (2).*

3. Scheme

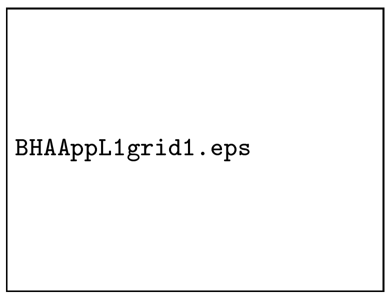
Notation 3.1. *Let* $\mathcal{T}$ *be a conform unstructured triangulation of* $I\!R^2$*, that is a partition in triangles, pairwise intersections of which are either empty, a common vertex or a common edge.*

- I *denotes the set of vertices in* $\mathcal{T}$*,*
- N_i *for* $i \in I$ *denotes the set of* i*'s neighbouring vertices,*
- M_{ij} *denotes the midpoint of the edge in* $\mathcal{T}$ *which connects* $i, j \in I$*,*
- $G^+_{ij}, (G^-_{ij})$ *denotes the center of gravity of the unique triangle in* $\mathcal{T}$ *which has vertices* $i, j, k \in I$ *and for which this enumeration is positively (negatively) oriented,*
- C_i *for* $i \in I$ *denotes the dual cell around* i*, that is the polygonal area with boundary* $\bigcup_{j \in N_i} \overline{G^+_{ij} M_{ij}} \cup \overline{G^-_{ij} M_{ij}}$*,*
- L_{ij} *for* $i \in I, j \in N_i$ *denotes the quadrangular area given by the convex hull of the vertices* i, G^-_{ij}, j, G^+_{ij}.

These cells C_i are taken as finite volume cells for the first step of the scheme, the cells L_{ij} are taken for the second step.



FV cells C_i for first step

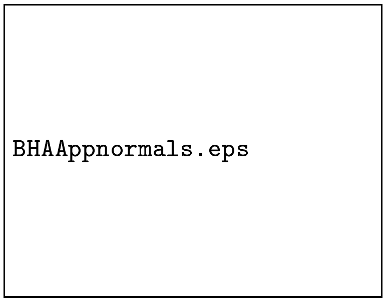


FV cells L_{ij} for second step

We need some further notations where $A(P)$ denotes the area of the Polygon $P \subset I\!R^2$.:

Notation 3.2. *For all* $i \in I, j \in N_i$ *we define*

- $\vec{\eta}^{\,+}_{ij}, (\vec{\eta}^{\,-}_{ij})$:= *outer scaled normal of* C_i *for the edge* $\overline{M_{ij} G^+_{ij}}, \left(\overline{M_{ij} G^-_{ij}}\right)$, *(that means* $\left|\vec{\eta}^{\,+}_{ij}\right| = \left|\overline{M_{ij} G^+_{ij}}\right|$*),*
- $\vec{\theta}_{ij} := \vec{\eta}^{\,+}_{ij} + \vec{\eta}^{\,-}_{ij}, \qquad r_{ij} := \frac{A(L_{ij} \cap C_i)}{A(C_i)}.$



Normals to C_i and L_{ij}

For our convergence proof we need a nondegeneracy-condition on the triangulation.

Definition 3.3. *A conform unstructured triangulation $\mathcal{T}$ will be called an* (a,b)**-nondegenerate triangulation**, *if the length of its edges are bounded, i.e. the supremum h of these exists, and the areas of all triangles $D \in \mathcal{T}$ are bounded by*

$$ah^2 < A(D) < bh^2.$$

Finally we define the discrete solution obtained by the staggered Lax-Friedrichs scheme. In [3] these formulas are obtained by applying evolution-projection ideas on the finite volume cells C_i and L_{ij}.

Definition 3.4 (StgLxF-scheme)**.** *Let $\mathcal{T}$ be an (a,b)-nondegenerate triangulation, $\Delta t > 0$. We define for all $i \in I, j \in N_i, n \in 2\mathbb{N}$*

$$u_i^0 := \frac{1}{A(C_i)} \int_{C_i} u_0, \tag{4}$$

$$u_{ij}^{n+1} := \frac{1}{2}\left(u_i^n + u_j^n\right) - \frac{\Delta t}{A(L_{ij})}\left(\vec{f}\left(u_j^n\right) - \vec{f}\left(u_i^n\right)\right)\vec{\theta}_{ij}, \tag{5}$$

$$u_i^{n+2} := \sum_{j \in N_i} r_{ij} u_{ij}^{n+1} - \frac{\Delta t}{A(C_i)} \sum_{j \in N_i} \vec{f}\left(u_{ij}^{n+1}\right)\vec{\theta}_{ij}. \tag{6}$$

These values define the numerical solution

$$u_h(x,t) := \begin{cases} u_i^n & for \quad (x,t) \in C_i^o \times [n\Delta t, (n+1)\Delta t) \\ u_{ij}^{n+1} & for \quad (x,t) \in L_{ij}^o \times [(n+1)\Delta t, (n+2)\Delta t) \end{cases}. \tag{7}$$

4. Convergence

Theorem 4.1 (Convergence of StgLxF-scheme)**.** *Let $f_1, f_2 \in C^1(\mathbb{R})$, $u_0 \in L^1\left(\mathbb{R}^2\right) \cap L^\infty\left(\mathbb{R}^2\right)$ with $B := \|u_0\|_{L^\infty}$ and $T > 0$. We define*

$$L := \max_{u \in [-B,B], i=1,2} |f_i'(u)|.$$

Let further $(\mathcal{T}_k)_{k\in\mathbb{N}}$ *be a sequence of uniformly* (a,b)*-nondegenerate triangulations in the sense of Def.3.3 (a,b independent of k) with* $\lim_{k\to\infty} h_k = 0$*. Let* β,γ *be constants with the relation* $0 < \gamma < \beta < \frac{a}{4}$*. Let* $(\Delta t_k)_{k\in\mathbb{N}}$ *be a sequence of timesteps, such that for all* $k \in \mathbb{N}$ *the CFL-condition* $\frac{\Delta t_k}{h_k} L \leq \beta$ *and the lower bound* $\gamma \leq \frac{\Delta t_k}{h_k} L$ *hold.* $(u_{h_k})_{k\in\mathbb{N}}$ *denotes the associated sequence of numerical solutions defined by the StgLxF-scheme (4) - (7).*

Then the sequence $(u_{h_k})_{k\in\mathbb{N}}$ *converges in* $L^q_{loc}\left(\mathbb{R}^2 \times [0,T]\right)$ *for all* $1 \leq q < \infty$ *strongly towards the unique weak entropy solution of the Cauchy-problem (1) and (2).*

Remark 4.1. *The proof of the theorem provides means which not only prove the statement, but allow to improve the convergence result to an a–priori error estimate of Kruzhkov-type* $\|u - u_h\|_{L^1} \leq Ch^{1/4}$*. This can be done following a paper of Bouchut and Perthame* [5]*. The result can also be generalized in several further directions. Similar to* [12] *convergence can be obtained for higher order schemes, in our case the whole family of NT-schemes* [3]*. Extensions to the case of weakly coupled systems* [15] *can be obtained with the same technique. The proof appears to be applicable even in the case of three space-dimensions by transferring the geometric dimension-dependent considerations. This would result in a convergence proof for the three-dimensional NT-schemes which were introduced by Arminjon on this conference* [1]*.*

The proof of Thm. 4.1 consists of showing that the conditions 2.2 a) to e) are satisfied by the sequence $(u_{h_k})_{k\in\mathbb{N}}$. The major steps of this will now be formulated in several propositions. As mentioned before, proofs can be found in [8]. The first properties are discrete maximum-principles, which imply a L^∞-bound.

Proposition 4.2 (Uniform L^∞-bound)**.** *Let the assumptions of Thm. 4.1 be valid,* u_h *be an element of the sequence of numerical solutions. Then we have for all* $n \in \mathbb{N}, i \in I, j \in N_i$ $\quad \left|u_i^0\right| \leq B$ *and*

$$\min\left\{u_i^n, u_j^n\right\} \leq u_{ij}^{n+1} \leq \max\left\{u_i^n, u_j^n\right\}, \quad \min_{j\in N_i}\left\{u_{ij}^{n+1}\right\} \leq u_i^{n+2} \leq \max_{j\in N_i}\left\{u_{ij}^{n+1}\right\}.$$

Therefore condition 2.2 a) is satisfied by $C_1 := B$*.*

Next we derive two discrete entropy inequalities.

Proposition 4.3 (Discrete entropy inequalities)**.** *Let the assumptions of Thm. 4.1 be valid,* u_h *be an element of the sequence of numerical solutions. Then for all* $n \in \mathbb{N}, i \in I, j \in N_i$ *and entropy pairs* $(U, \vec{F})$ *hold*

$$U\left(u_{ij}^{n+1}\right) \leq \frac{1}{2}\left(U\left(u_i^n\right) + U\left(u_j^n\right)\right) - \frac{\Delta t}{A\left(L_{ij}\right)}\left(\vec{F}\left(u_j^n\right) - \vec{F}\left(u_i^n\right)\right)\vec{\theta}_{ij}, \quad (8)$$

$$U\left(u_i^{n+2}\right) \leq \sum_{j\in N_i} r_{ij} U\left(u_{ij}^{n+1}\right) - \frac{\Delta t}{A\left(C_i\right)}\sum_{j\in N_i}\vec{F}\left(u_{ij}^{n+1}\right)\vec{\theta}_{ij}. \quad (9)$$

This Proposition turns out to be the most important point in the proof, as it is fundamental for all further results. The problem of the initially unknown structure of the entropy inequalities turns out to be perfectly solved by the inequalities (8),(9): first these inequalities are actually satisfied by the scheme and secondly they allow to derive the further properties which are sufficient for convergence of the scheme. From these entropy inequalities follow uniform L^1- and L^2-stability.

Proposition 4.4 (Uniform L^1- and L^2-stability). *Let the assumptions of Thm. 4.1 be valid, u_h be an element of the sequence of numerical solutions. Then* $\|u_h(\cdot,0)\|_{L^1(I\!R^2)} \leq \|u_0\|_{L^1(I\!R^2)}, \quad \|u_h(\cdot,0)\|_{L^2(I\!R^2)} \leq \|u_0\|_{L^2(I\!R^2)} \leq \infty$ *and for all* $0 \leq t_1 \leq t_2 \leq T$

$$\|u_h(\cdot,t_2)\|_{L^1(I\!R^2)} \leq \|u_h(\cdot,t_1)\|_{L^1(I\!R^2)}, \quad \|u_h(\cdot,t_2)\|_{L^2(I\!R^2)} \leq \|u_h(\cdot,t_1)\|_{L^2(I\!R^2)}$$

hold. Therefore conditions 2.2 b) and e) are satisfied by $C_2 := \|u_0\|_{L^1(I\!R^2)}$.

For the proof of the remaining conditions 2.2 c) and d) we need a weak BV-estimate. This is based on a more accurate estimate of the entropy dissipation for a quadratic entropy.

Lemma 4.5 (Entropy dissipation). *Let the assumptions of Thm. 4.1 be valid. Then there exists a constant $C > 0$ such that for all numerical solutions u_h of the sequence, all $n \in I\!N, i \in I, j \in N_i$ and entropy pairs $(U, \vec{F})$ with $U(u) = u^2/2$ holds*

$$U\left(u_{ij}^{n+1}\right) - \frac{1}{2}\left(U(u_i^n) + U\left(u_j^n\right)\right) + \frac{\Delta t}{A\left(L_{ij}\right)}\left(\vec{F}\left(u_j^n\right) - \vec{F}(u_i^n)\right)\vec{\theta}_{ij} \leq -C\left(u_i^n - u_j^n\right)^2.$$

Using this estimate, we derive a weak BV-estimate. This kind of estimate is weaker than a BV-estimate, but strong enough to obtain convergence. A similar estimate was derived in the proof for the case of the linear equation [4], called an estimate on the weighted total variation. We denote $I_D := I \cap D$ as the set of vertices in D.

Proposition 4.6 (Weak BV-estimate). *Let the assumptions of Thm. 4.1 be valid, $D \subset I\!R^2$ be a disc. Then there exists a constant C, such that for all numerical solutions u_h of the sequence and corresponding $h, \Delta t$ with $N := max\{n \in 2I\!N | n\Delta t \leq T\}$ holds*

$$\sum_{\substack{i \in I_D, j \in N_i \\ n \in 2I\!N, n \leq N}} h^2 \left|u_i^n - u_j^n\right| \leq C h^{-\frac{1}{2}}.$$

With this weak BV-estimate we can show that the weak consistency estimates Thm. 2.2 c) and d) are satisfied by $\kappa := 1/2$, cf. [8].

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