The lifespan estimate for 1D semilinear wave equations with spatial weights and compactly supported data

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In this talk, I will discuss about initial value problems for semilinear wave equations with spatial weights in one space dimension. The lifespan estimates of classical solutions for compactly supported data are established in all the cases of polynomial weights. The results are classified into two cases according to the total integral of the initial speed.

$u = u(x,t)$

$$
(P)\begin{cases} u_{tt} - u_{xx} = \dfrac{|u|^p}{(1+x^2)^{\frac{1+a}{2}}} & \text{in } \mathbb{R}\times(0,\infty) \\ u|_{t=0} = \varepsilon f, \quad u_t|_{t=0} = \varepsilon g \end{cases}
$$

$p>1,\ a\in\mathbb{R},\ 0<\varepsilon\ll 1$

$(f,g)\in C_0^2\times C_0^2(\mathbb{R})$

$T(\varepsilon)$: lifespan of classical sol of (P)

Motiv

$$
\frac{|u|^p}{(1+t)^{p-1}} \xleftarrow{u=(1+t)v} v_{tt} - v_{xx} + \frac{2}{1+t}v_t = |v|^p
$$

※ $\exists p$: global sol

Thm (K. Morisawa Takamura)

Ⓐ $\int_{\mathbb{R}} g\,dx \neq 0$

$$
T(\varepsilon) \sim \begin{cases} C\varepsilon^{-\frac{p-1}{1-a}} & (a<0) \\ \phi^{-1}(C\varepsilon^{-(p-1)}) & (a=0) \\ C\varepsilon^{-(p-1)} & (a>0) \end{cases}
$$

$\phi(s) = s\log(2+s)$

Ⓑ $\int_{\mathbb{R}} g\,dx = 0$

$$
T(\varepsilon) \sim \begin{cases} C\varepsilon^{-\frac{p(p-1)}{1-ap}} & (a<0) \\ \psi^{-1}(C\varepsilon^{-p(p-1)}) & (a=0) \\ C\varepsilon^{-p(p-1)} & (a>0) \end{cases}
$$

$\psi(s) = s\log^p(2+s)$

Ref

- $a=-1$ by Zhou (1992)
- $a>-1$ f,g: non-compact & decay

  $|u|^p \rightsquigarrow |u|^{p-1}u$

  Suzuki (2010)

  Kubo, Osaka, Yazici (2013)

  … f,g: odd, $ap>1 \Rightarrow$ global sol
- $a>-1$, $f\in L^\infty$ $g\in L^1$

  Wakasa (2017)

  … $T(\varepsilon)$ of (A)

Outline of proof

$\mathrm{supp}\{f,g\}\subset[-R,R]$ $(R>1)$

$$(P) \Longleftrightarrow u=\underbrace{\varepsilon u^0}_{\text{linear}}+\underbrace{L_a(|u|^p)}_{\text{Duhamel}}$$

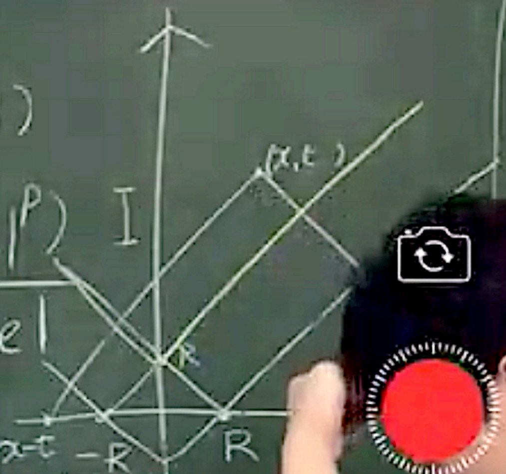
$(x,t)\in \mathrm{I}$

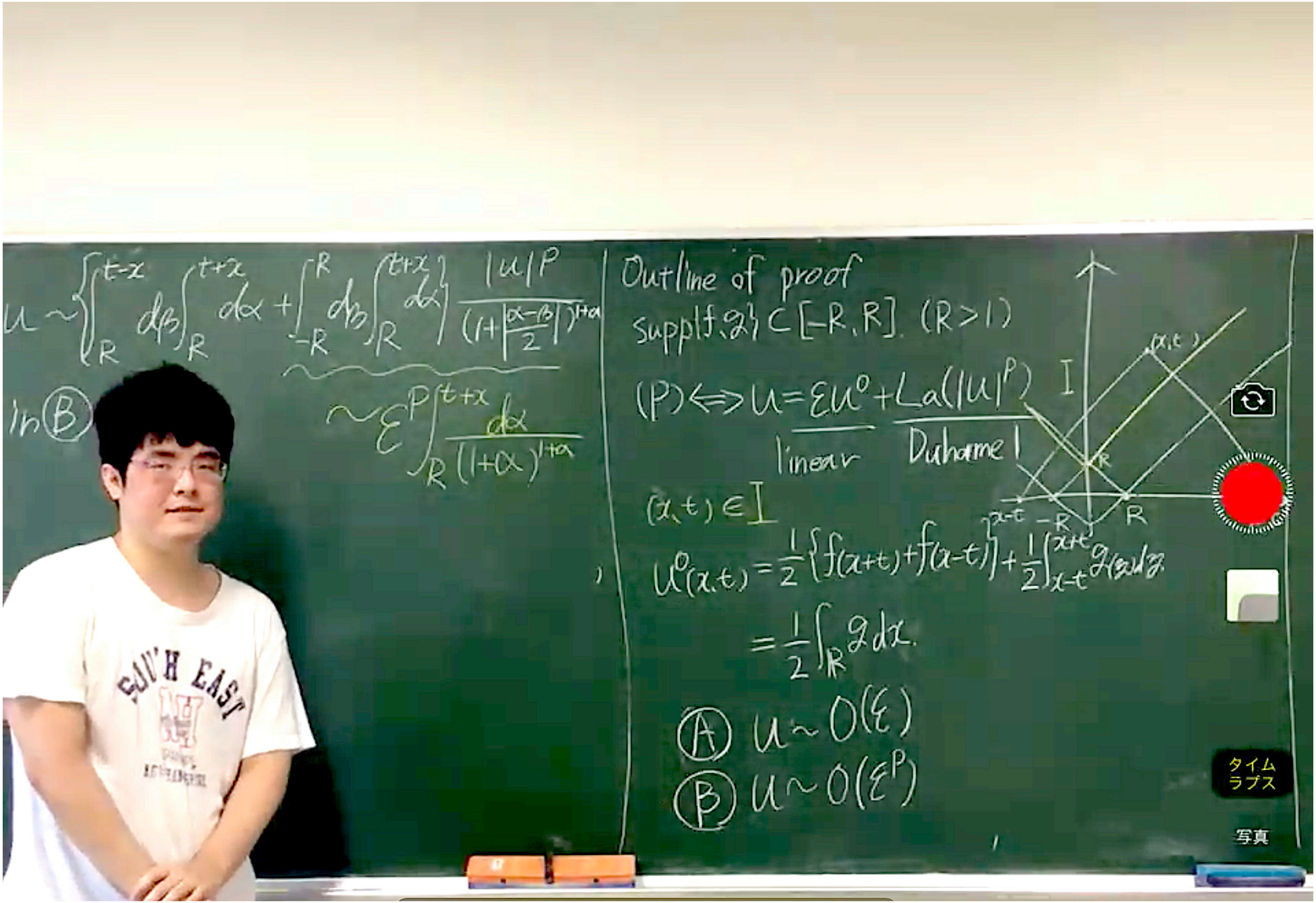
$$u^0(x,t)=\frac{1}{2}\{f(x+t)+f(x-t)\}+\frac{1}{2}\int_{x-t}^{x+t}g(y)dy$$

$$=\frac{1}{2}\int_{\mathbb{R}} g\,dx.$$

(A) $u\sim O(\varepsilon)$

(B) $u\sim O(\varepsilon^p)$





$u \sim \left\{ \int_R^{t-x} d\beta \int_R^{t+x} d\alpha + \int_{-R}^{R} d\beta \int_R^{t+x} d\alpha \right\} \frac{|u|^p}{\left(1+\left|\frac{\alpha-\beta}{2}\right|\right)^{1+\alpha}}$
In (B)
$\sim \varepsilon^p \int_R^{t+x} \frac{d\alpha}{(1+\alpha)^{1+\alpha}}$
Outline of proof
$\operatorname{supp}\{f, g\} \subset [-R, R]$. $(R > 1)$
$(P) \Longleftrightarrow u = \varepsilon u^0 + L_a(|u|^p)$
linear
Duhamel
I
(x,t)
$x-t$
$-R$
R
$(x,t) \in I$
$u^0(x,t) = \frac{1}{2}\{f(x+t)+f(x-t)\} + \frac{1}{2}\int_{x-t}^{x+t} g(y)\,dy$
$= \frac{1}{2}\int_R g\,dx$
(A) $u \sim O(\varepsilon)$
(B) $u \sim O(\varepsilon^p)$
タイム
ラプス
写真