Reconstruction of a solely time-dependent source in a time-fractional diffusion equation with non-smooth solutions

Karel Van Bockstal

Gent University
Belgium
karel.vanbockstal@ugent.be

Joint work with: A. S. Hendy

In this talk, an inverse source problem (ISP) for a time fractional diffusion equation of order $\alpha \in (0,1)$ where the coefficients of the elliptic operator are dependent on spatial and time variables is discussed. The missing solely time-dependent source is recovered from an additional integral measurement. First, the uniqueness of a solution to the ISP will be shown. Next, two numerical algorithms based on Rothe's method over uniform and graded grids will be proposed and the convergence of iterates towards the exact solution will be discussed. An essential feature of the fractional subdiffusion problem is that the solution lacks the smoothness near the initial time, although it would be smooth away from $t = 0$. Rothe's method on a uniform grid addresses the existence of a such a solution (non-smooth with t^γ term where $1 > \gamma > \alpha$) under low regularity assumptions, whilst Rothe's method over graded grids has the advantage to cope better with the behaviour at $t = 0$ (also here t^α is included in the class of admissible solutions) for the considered problems. This theoretical obtained results will be supported by numerical experiments.