Global dynamics for the critical Hardy-Sobolev parabolic equation below the ground state

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We consider the Cauchy problem of the energy critical Hardy-Sobolev parabolic equation

$$\begin{cases} \partial_t u - \Delta u = |x|^{-\gamma}|u|^{2^*(\gamma)-2}u, & (t,x) \in (0,T) \times \mathbb{R}^d, \\ u(0) = u_0 \in \dot{H}^1(\mathbb{R}^d), \end{cases}$$

where $d \geq 3$, $T > 0$, $\gamma \in [0,2)$, and $2^*(\gamma)$ is the critical Hardy-Sobolev exponent, i.e.,

$$2^*(\gamma) := \frac{2(d-\gamma)}{d-2}.$$

Our aim is to give a necessary and sufficient condition on initial data below or at the ground state, under which the behavior of solutions is completely dichotomized. More precisely, the solution exists globally in time and its energy decays to zero in time, or it blows up in finite or infinite time. The proof of global existence is based on the linear profile decomposition, the perturbative result, and the rigidity argument. The proof of blow up result is based on Levine's concavity method with some modifications. The result on the dichotomy for the corresponding Dirichlet problem is also shown as a by-product via a comparison principle.