On an explicit solution of an inverse problem for the time-fractional diffusion equation

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We consider the following inverse problem of finding a pair (u, p) for the time-fractional diffusion equation:

$$\Diamond_{\alpha,t} u = \partial_t^\alpha u(t,x) - \Delta u(t,x) = p(t) f(t,x) \text{ in } \Omega \times (0,T), \qquad (1)$$

$$u(0,x) = 0,\ x \in \Omega,$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with the boundary $\partial\Omega \in C^{1+\gamma}$, $0 < \gamma < 1$, $\Delta = \sum_{i=1}^n \partial_{x_i}^2$ is the Laplacian and

$$\partial_t^\alpha u(t,x) = \frac{1}{\Gamma(1-\alpha)} \int_0^t (t-\tau)^{-\tau} u'_\tau(\tau,x) d\tau$$

is the fractional Caputo time derivative of order $0 < \alpha \leq 1$. Here Γ is the gamma function. We shall note that for $\alpha = 1$ the fractional derivative coincides with the standard time derivative.

In this talk, our main goal is to present explicit solutions of inverse problems of recovering the time-dependent control function $p(t)$ in the Cauchy problem for the multidimensional time-fractional diffusion equation (1).

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